A specification (by Atomic Energy of Canada Limited) covering the minimum requirements for the seismic analysis of nuclear power plant equipment, based on maximum expected accelerations (show in floor response spectra) is discussed. Two numerical examples of nuclear pressure vessels are included.

The requirements of the National Building Code of Canada (minimum requirements for buildings and industrial installations) are demonstrated on the stability and safety of tall, free standing vertical pressure vessels such as refinery fractionators, etc.

TABLE OF CONTENT

Page

LIST OF SYMBOLS AND ABBREVIATIONS

A = Cross section area; in^2 (m^2)

A_1 = Base plate or saddle area; in^2 (m^2)

A_r = Acceleration ratio

A_s = Effective bolt area; in^2 (m^2)

b = Saddle width; in (m)

d = Saddle length; in (m)

D = Outside shell diameter; ft (m)

e = Excentricity; in (m)

E = Constant modulus of eleasticity; lb/in^2 (P_a)

E_1 = Joint efficiency

f_c = Compressive strength; lb/in^2

f_n = Natural frequency; Hz

F = Seismic lateral force; lb (N)

F_1, F_2, F_3 = Force reactions; lb (N)

F_b = Foundation factor

F_e = Portion of lateral seismic force applied at the centroid of a vessel; lb (kg)

F_t = Portion of lateral seismic force applied at the top of a vessel; lb (kg)

g = Acceleration due to gravity; in/sec^2 (m/s^2)

h = Shell thickness; in (m)

I = Moment of inertia; in^4 (m^4)

I_b = Importance factor = 1.0 for vessels in general

I_x = Moment of inertia around axis x-x; in^4 (m^4)

I_y = Moment of inertia around axis y-y; in^4 (m^4)

J = Numerical reduction coefficient of base overturning moment

k' = Spring stiffness; lb/in (kg/m)

k = Non dimentional parameter = $h^2/12R^2$

K = Numerical coefficient that reflects the material and type of construction

ℓ = Length of a bar or cantilever beam or a shell; in (m)

L = Height of a vessel; ft (m)

m = Mass of vessel or equipment; lb (kg)

m' = Mass per unit length

M = Bending moment at the base; in-lb (m-kg)

M_x = Bending moment at level x; in-lb (m-kg)

N_x = Longitudinal membrane force; lb (N)

P, P_1, P_2 = Load or vertical force; lb (N)

p = Distributed pressure load or design pressure; lb/in² (Pa)

Q = Seismic vertical force; lb (N)

R = Shell radius; in (m)

s = Lever length; in (m)

S = Seismic coefficient

S_a = Allowable stress; lb/in² (Pa)

S_b = Bending stress; lb/in² (Pa)

S_c = Compression stress; lb/in² (Pa)

S_p = Pressure stress; lb/in² (Pa)

S_t = Combined stresses; lb/in² (Pa)

t = Base plate thickness; in (m)

T = Fundamental period; sec (s)

T_{max} = Maximum kinetic energy

U_{max} = Maximum potential energy

V = Shear moment; lb (kg)

V_b = Minimum lateral seismic force at the base of the vessel; lb (N)

w = Distributed load; lb/in (kg/m)

W = Total weight of vessel; lb (kg)

x = Level; in (m)

y = Deflection; in (m)

y_o = Static deflection; in (m)

Z = Section modulus; in³ (m³)

ν = Poisson's ratio

$\lambda = \frac{\pi R}{\ell}$ = constant

$\lambda_m = \varepsilon_m R/\ell$

ε_m = Constant

β = Damping coefficient

ρ = Mass density

ω = Frequency of free vibration; radian/sec (rad/s)

ω_1 = Lowest natural frequency; radian/sec (rad/s)

Ω = Frequency parameter

θ = Slope of the elastic curve = $\frac{dy}{dx}$

1.0 INTRODUCTION

Considerable knowledge has been gained in the last three decades about the phenomena of ground motion, the characteristics of structures such as pressure vessels, and their behavior in earthquakes. In addition, much has been learned about the response of various vibrating systems to such motion. Despite this progress and coincidental development of earthquake design criteria and codes, the unknowns and the complexities are still so great that earthquake resistant design is not yet capable of complete and rigorous execution solely by means of mathematical analysis, design codes, specifications, or rules of procedure. It is an art as well as a science, and requires experience and judgment on the part of the engineer, as well as sensitivity to the true nature of the problem including the behavior of materials and structures subject to various types and degrees of motion. Above all it is necessary to have an understanding of the manner in which a pressure vessel absorbs the energy transmitted to it by an earthquake and the maximum amount of motion or energy the pressure vessel can sustain.

It is intended that this report will furnish current information pertaining to these topics and specifically to the earthquake resistant design of pressure vessels.

The problem involves more than merely achieving an adequate design. The objectives of the design must be attained in the actual construction of the pressure vessel. The development of design specifications and construction procedures for earthquake-resistant structures has been and, in fact, still is an evolutionary process. Although most design specifications or codes involve the concept of a statically equivalent lateral design force, the appropriate choice of the equivalent static force is governed by the dynamic behavior of the pressure vessel. The design of earthquake-resistant pressure vessels is basically a dynamic and not a static problem. For a working understanding of the problem, one must consider inelastic deformation and energy absorption and must take into account the period of vibration of the pressure vessel and nature of the resistance of the pressure vessel under all conditions to which it is likely to be subjected.

The net result is that while the more or less rigorous analyses are very helpful in guiding the establishment of design criteria, practical design criteria at this time are simplifications of the complex dynamic phenomena into equivalent static criteria. From a practical standpoint, earthquake resistance developed consistently under simplified elastic criteria, using static equivalent forces to provide a simulated envelope of the dynamic forces, shears, and moments, is probably of greater importance than highly analytical solutions [1], [2], [3], [12].

2.0 NATURAL FREQUENCIES OF PRESSURE VESSELS

The natural frequency of vibration is an important characteristic of tall vertical and long horizontal vessels, required in all seismic analysis methods.

Vertical or horizontal vessel can be represented as beams, which could have several end support configuration such as:

Simply supported

Cantilever

Clamped - clamped

Clamped - hinged

This part will cover various methods for the computation of the natural frequency and the fundamental period of vibration, for the above mentioned conditions [4], [5].

2.1 Axial vibrations at a bar of uniform cross section

With negligible weight, loaded by a force "P" at one end, the other end being fixed, (fig. 2.1.1), the natural frequency is given by

$$\omega_n = \sqrt{\frac{k'}{m}} \text{ in rad/sec} \qquad (2.1.1)$$

$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{k'}{m}} = \frac{1}{2\pi}\sqrt{\frac{k'g}{P}} \text{ in Hz} \qquad (2.1.2)$$

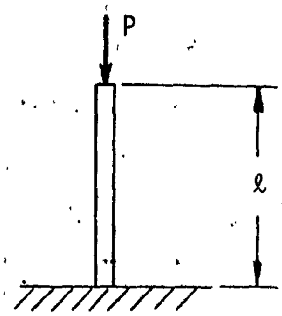


Figure 2.1.1

Where $k' = \frac{EA}{\ell}$ = spring stiffness (2.1.3)

P = load or force; lb. (N)

A = cross section area; in^2 (m^2)

E = modulus of elasticity; lb/in^2 (Pa)

g = acceleration due to gravity; in/sec^2 (m/s^2)

ℓ = length of the bar; in (m)

The fundamental period, T, is

$$T = \frac{1}{f_n} \qquad (2.1.4)$$

2.2 Lateral vibrations of a cantilever beam

With a weight W mounted upon the end (fig. 2.2.1). The equation for the static deflection y_o of the beam, due to the weight on the end, is obtained from any book of strength of materials and is

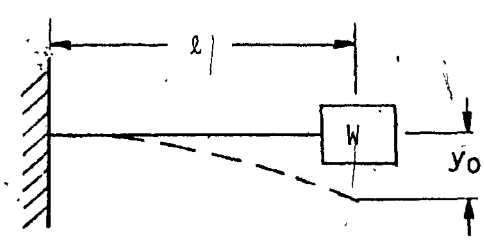


Figure 2.2.1

$$y_o = \frac{W\ell^3}{3EI} \qquad (2.2.1)$$

Where ℓ = length of the beam

E = modulus of elasticity

I = flexural moment of inertia

W = weight of mass (m)

y_o = static deflection

The spring stiffness k' is the force required to produce unit deflection, eq. (2.2.1) can be rearranged to give

$$k' = \frac{W}{y_o} = \frac{3EI}{\ell^3} \qquad (2.2.2)$$

Substituing this value for k' in the equation for natural frequency, eq. (2.1.1) produces

$$\omega n = \sqrt{\frac{k'}{m}} = \sqrt{\frac{3EI/\ell^3}{W/g}} = \sqrt{\frac{3EIg}{W\ell^3}} \text{ in rad/sec} \qquad (2.2.3)$$

$$f_n = \frac{1}{2\pi} \sqrt{\frac{3EIg}{W\ell^3}} \text{ in Hz} \qquad (2.2.4)$$

Fundamental period T

$$T = \frac{1}{f_n} \text{ in sec} \qquad (2.2.5)$$

Note: For any single-degree-of-freedom system, in which the quanty W/k' represents the static deflection of a spring due to the weight W acting upon it, the natural-frequency equation yields, by substituing $m = W/g$:

$$\omega n = \sqrt{\frac{k'}{m}} = \sqrt{\frac{g}{W/k'}} = \sqrt{\frac{g}{y_o}} \qquad (2.2.6)$$

Substituing the value of y_0 from eq. (2.2.1)

$$\omega_n = \sqrt{\frac{g}{y_0}} = \sqrt{\frac{g}{W\ell^3/3EI}} = \sqrt{\frac{3EIg}{W\ell^3}} \qquad (2.2.7)$$

which is the same obtained in eq. (2.2.3)

Example 2.1 A vertical pressure vessel as shown in fig. (2.2.2) is supported on 3 legs and its weight is 1000 lbs. Each leg is made by a 4" SCH 80 pipe and is anchored on a concrete base. What is the natural frequency and the fundamental period of one leg in the vertical and horizontal axis.

SOLUTION

4" SCH 80 pipe

cross section area $A = 4.41\ in^2$

moment of inertia $I = 9.61\ in^4$

a) Vertical axis

For a bar of uniform cross section loaded by a force "P" at one end, the other being fixed, the natural frequency is given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k'g}{P}}$$

where spring stiffness $K' = \frac{EA}{\ell}$

$$k' = \frac{30 \times 10^6 \times 4.41}{27} = 4.9 \times 10^6\ lb/in$$

and $f_n = \frac{1}{2\pi}\sqrt{\frac{4.9 \times 10^6 \times 386}{333}} = \underline{\underline{379}}\ hz$

Fundamental period T

$$T = \frac{1}{f_n} = \frac{1}{379} = \underline{\underline{0.0026\ sec}}$$

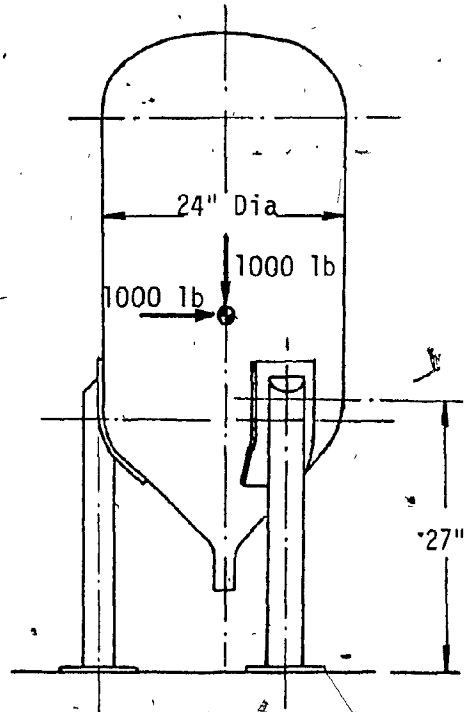


Figure 2.2.2

b) *Horizontal axis*

Assume 1/3 of total weight applied horizontally to the end of a cantilever (fig. 2.2.3)

$$P = \frac{1}{3}(1000) = 333 \text{ lbs}$$

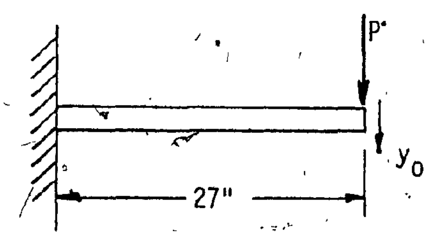


Figure 2.2.3

For a cantilever beam of uniformed cross section, loaded by a force "P" as shown at one end, and fixed at the other end, the statis deflection y_o *is:*

$$y_o = \frac{W\ell^3}{3EI}$$

and natural frequency is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{k'}{m}} = \sqrt{\frac{g}{y_o}}$$

$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{3EIg}{W\ell^3}} = \frac{1}{2\pi}\sqrt{\frac{3(30\times10^6)\ (9.61)\ (386)}{333\ (27^3)}}$$

$$\underline{\underline{f_n = 36\ Hz}}$$

Fundamental period T

$$T = \frac{1}{f_n} = \frac{1}{36} = \underline{\underline{0.028\ sec.}}$$

2.3 Differential equation for lateral vibration of beams

In order to determine the differential equation for the lateral vibration of beams, the Euler equation for the beam can be used [4].

Consider the forces and moments acting on an element of the beam shown in fig. (2.3.1)

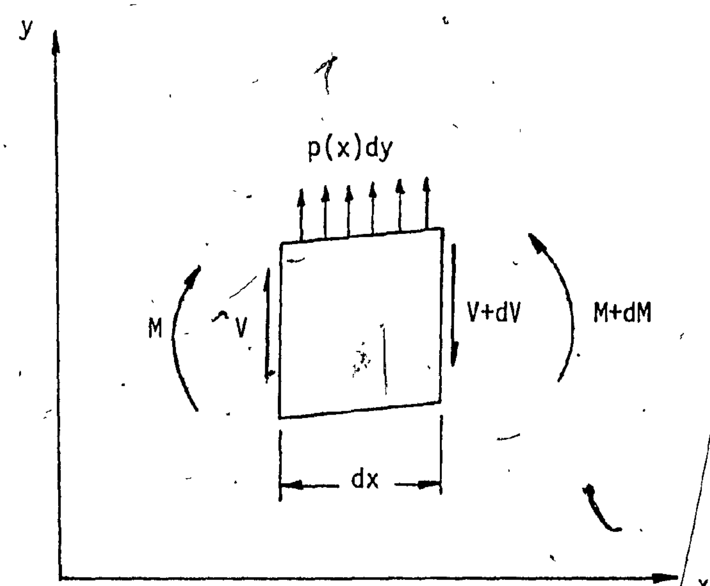


Figure 2.3.1

V and M are shear and bending moments, respectively, and p(x) represents the loading per unit length of the beam. By summing forces in the y direction.

$$dV - p(x)\,dx = 0 \tag{2.3.1}$$

By summing moments about any points on the right face of the element

$$dM - V\,dx - 1/2p\,(x)\,(dx)^2 = 0 \tag{2.3.2}$$

In the limiting process these equations result in the following important relationships

$$\frac{dV}{dx} = p(x), \quad \frac{dM}{dx} = V \tag{2.3.3.}$$

The first part of eq. (2.3.3) states that the rate of change of shear along the length of the beam is equal to the loading per unit length, and the second states that the rate of change of the moment along the beam is equal to the shear.

From eq. (2.3.3) we obtain the following

$$\frac{d^2M}{dx^2} = \frac{dV}{dx} = p(x) \tag{2.3.4}$$

The bending moment is related to the curvature by the flexure equation, which, for the coordinates indicated in fig. (2.3.1) is

$$M = EI\,\frac{d^2y}{dx^2} \tag{2.3.5}$$

Substituting this relation into eq. (2.3.4), we obtain

$$\frac{d^2}{dx^2}\left(EI\,\frac{d^2y}{dx^2}\right) = p(x) \tag{2.3.6}$$

For a beam vibrating about its static equilibrium position under its own weight, the load per unit length is equal to the inertia load due to its mass and acceleration. Since the inertia force is in the same direction as p(x) as shown in fig. (2.3.1), we have, by assuming harmonic motion

$$p(x) = \frac{W}{g} \omega^2 y \tag{2.3.7}$$

Where $\frac{W}{g}$ is the mass per unit length of the beam. Using this relation, the equation for the lateral vibration of the beam reduces to

$$\frac{d^2}{dx^2} \left(EI \frac{d^2y}{dx^2}\right) - \frac{W}{g} \omega^2 y = 0 \tag{2.3.8}$$

In the special case where the flexural rigidity EI is a constant, the above equation may be written as

$$EI \frac{d^4y}{dx^4} - \frac{W}{g} \omega^2 y = 0 \tag{2.3.9}$$

On substituing

$$k^4 = \frac{W}{g} \frac{\omega^2}{EI} \tag{2.3.10}$$

We obtain the fourth-order differential equation

$$\frac{d^4y}{dx^4} - k^4 y = 0 \tag{2.3.11}$$

for the vibration of a uniform beam.

The general solution of eq. (2.3.11) can be shown to be

$$y = A \cosh kx + B \sinh kx + C \cos kx + D \sin kx \tag{2.3.12}$$

To arrive at this result, we assume a solution of the form

$$y = e^{ax}$$

which will satisfy the diffential equation when

$a = \pm k$ and $a = \pm ik$

Since

$$e^{\pm kx} = \cosh kx \pm \sinh kx$$

$$e^{\pm ikx} = \cos hx \pm i \sin kx$$

the solution in the form of eq. (2.3.12) is readily established. The natural frequencies of vibration are found from eq. (2.3.10) to be

$$\omega_n = k^2 \sqrt{\frac{g\ EI}{W}} \qquad (2.3.13)$$

Where the number k depends on the boundary conditions of the problem.

For a uniformed beam clamped at one end and free at the other the natural frequencies of vibration can be determined as follow:

The boundary conditions are

$$\text{at } x = 0 \begin{cases} y = 0 \\ \frac{dy}{dx} = 0 \end{cases}$$

$$\text{at } x = \ell \begin{cases} M = 0 \text{ or } \frac{d^2y}{dx^2} = 0 \\ V = 0 \text{ or } \frac{d^3y}{dx^3} = 0 \end{cases}$$

Substituing these boundary conditions in the general solution, we obtain

$$(y)_{x=0} = A + C = 0 \qquad \therefore A = -C$$

$$\left(\frac{dy}{dx}\right)_{x=0} = k\,[A \sinh 0 + B \cosh 0 - C \sin 0 + D \cos 0]_{x=0} = 0$$

$$k\,[B + D] = 0, \quad \therefore B = -D$$

$$\left(\frac{d^2y}{dx^2}\right)_{x=\ell} = k^2\,[A \cosh k\ell + B \sinh k\ell - C \cos k\ell - D \sin k\ell] = 0$$

$$A\,(\cosh k\ell + \cos k\ell) + B\,(\sinh k\ell + \sin k\ell) = 0$$

$$\left(\frac{d^3y}{dx^3}\right)_{x=\ell} = k^3\,[A \sinh k\ell + B \cosh k\ell + C \sin k\ell - D \cos k\ell] = 0$$

$$A\,(\sinh k\ell - \sin k\ell) + B\,(\cosh k\ell + \cos k\ell) = 0$$

From the last two equations we obtain

$$\frac{\cosh k\ell + \cos k\ell}{\sinh k\ell - \sin k\ell} = \frac{\sinh k\ell + \sin k\ell}{\cosh k\ell + \cos k\ell}$$

Which reduces to

$$\cosh k\ell \cos k\ell + 1 = 0 \qquad (2.3.14)$$

This last equation is satisfied by a number of values of $k\ell$, corresponding to each normal mode of oscillation, which are the positive roots of eq. (2.3.14)

$$k_n\ell = (1.875,\ 4.695,\ 7.854,\ \ldots) \qquad (2.3.15)$$

∴ The lowest natural frequency for the first mode is hence given as

$$\omega_1 = \frac{1.875^2}{\ell^2}\sqrt{\frac{gEI}{W}} = \frac{3.515}{\ell^2}\sqrt{\frac{gEI}{W}} \ \text{(rad/sec)} \qquad (2.3.16)$$

The following table lists numerical values of $(k\ell)^2$ for typical end conditions.

Beam configuration	$(k_1\ell)^2$ Fundamental	$(k_2\ell)^2$ Second mode	$(k_3\ell)^2$ Third mode
Simply supported	9.87	39.5	88.9
Cantilever	3.52	22.4	61.7
Clamped-hinged	15.4	50.0	104.0
Clamped-clamped	22.4	61.7	121.0

From eq. (2.3.16)

$$f_n = \frac{\omega_1}{2\pi} = \frac{3.515}{\ell^2 2\pi}\sqrt{\frac{gEI}{W}} \text{ in cps} \tag{2.3.17}$$

Fundamental period T

$$T = \frac{1}{f_n}$$

This method is limited to very simple vertical vessels with constant wall thickness and constant weight distribution over the entire length of the vessel.

Example 2.2 A fractionator tower is 36" diameter and 73 feet heigh. The tower material is SA-516, Gr. 70, and the thicknesses for the pressure part and skirt are identical (7/16"). The tower operates at 350°F. The operating weight is 64000 lbs. Find the fundamental period of the tower when erected?

SOLUTION

Assuming the following uniform cantilever beam conditions shown in fig. 2.3.2

$$w = \frac{64000}{73 \times 12} = 73 \ lb/in$$

For carbon steel at $350^{0}F$

$$E \approx 30\ 000\ 000 \ psi$$

For thin wall vessel

$$I = \pi R^3 h$$

$$\therefore I = \pi (36)^3 \ (.437) = 64053 \ in^4$$

using eq. (2.3.16)

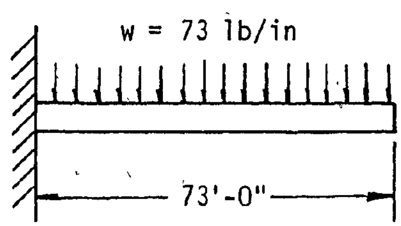


Figure 2.3.2

the natural frequency for uniform cantilever beam is

$$\omega_n = \frac{3.515}{\ell^2} \sqrt{\frac{gEI}{w}}$$

$$\omega_n = \frac{3.515}{(73 \times 12)^2} \sqrt{\frac{(386) \ (30000000) \ (64053)}{73}}$$

$$\omega_n = 14.60 \ rad/sec.$$

$$f_n = \frac{\omega_n}{2\pi} = \frac{14.60}{2\pi} = 2.32\ Hz$$

Fundamental period T

$$T = \frac{1}{f_n} = \frac{1}{2.32} = \underline{\underline{0.430\ sec}}$$

2.4 Rayleigh's method

In the case of vibrating beams there may be many masses involved or the mass may be distributed. A separate differential equation must be written for each mass or element of mass of any system and these equations solved simultaneously if we are to obtain the equations of motion of any multimass system. It is clear that the mathematics of such an approach would be overwhelming.

Here we shall present an energy method, which is due to Lord Rayleigh, because of its importance in the study of vibration [5].

It is probable that the equation

$$\omega_n = \sqrt{\frac{k'}{m}} = \sqrt{\frac{g}{W/k'}} = \sqrt{\frac{g}{y_0}}$$

first suggested to Rayleigh the idea of employing the static deflection to find the natural frequency of a system.

If we consider a freely vibrating system without damping, then, during motion, no energy is added to the system nor is any taken away. Yet when the mass has velocity, kinetic energy exists, and when the spring is compressed or extended, potential energy exists. Since no energy is added or taken away, the maximum kinetic energy of a system must be the same as the maximum potential energy. This is the basis of Rayleigh's method;

$$T_{max} = U_{max} \tag{2.4.1}$$

In order to see how it works let us apply the method to the simple system of fig. (2.2.1) of this report. Assuming that the motion is harmonic and of the form

$$y = y_0 \sin \omega_n t \tag{2.4.2}$$

The potential energy is a maximum when the spring is fully extended or compressed and occurs when $\sin \omega_n t = 1$.

$$\therefore U_{max} = \frac{W y_0}{2} \tag{2.4.3}$$

The velocity of the weight is given by

$$\frac{dy}{dt} = \dot{y} = y_0 \, \omega_n \cos \omega_n t \tag{2.4.4}$$

The kinetic energy reaches a maximum when the velocity is a maximum, that is, when $\cos \omega_n t = 1$

$$\therefore T_{max} = \frac{1}{2} m v_{max}^2 = \frac{W}{2g} (y_0 \omega_n)^2 \tag{2.4.5}$$

Equating the kinetic and potential energies, the fundamental frequency of the system is determined

$$T_{max} = U_{max}$$

$$\therefore \frac{W}{2g} (y_0 \omega_n)^2 = \frac{W y_0}{2}$$

$$y_0^2 \omega_n^2 = y_0 g$$

$$\omega_n = \sqrt{\frac{g}{y_0}} \tag{2.4.6}$$

Which is identical with equation (2.2.3).

Rayleigh showed that this procedure can also be applied to systems of higher degrees of freedom provided a reasonable distribution of the deflection is assumed.

It is true in multimass systems that the dynamic deflection curves are not the same as the static deflection curves. The importance of the method, however, is that any reasonable deflection curve can be used in the process. The static deflection curve is a reasonable one, and hence it gives a good approximation. Rayleigh also has shown that the correct curve will always give the lowest value for the natural frequency. This is to say, that if many deflection curves are assumed, the one giving the lowest natural frequency is the best.

2.4.1 Simply beam vibrations

In this section we wish to extend Rayleigh's method to beam vibrations [4].

Letting m be the mass per unit length along the beam and y the amplitude of the assumed deflection curve, the kinetic energy is expressed by the equation.

$$T_{max} = \frac{1}{2}\,\omega^2 \int_0^{\ell} m y^2 \, dx \qquad (2.4.7)$$

The potential energy of the beam is determined by the work done on the beam which is stored as elastic energy.

Letting M be the bending moment and θ the slope of the elastic curve, the work done is equal to

$$U = \frac{1}{2} \int M \, d\theta$$

Since the deflection in beams is generally small, the following geometric relations are assumed to hold, fig. (2.4.1)

$$\theta = \frac{dy}{dx}, \quad \frac{1}{R} = \frac{d\theta}{dx} = \frac{d^2y}{dx^2}$$

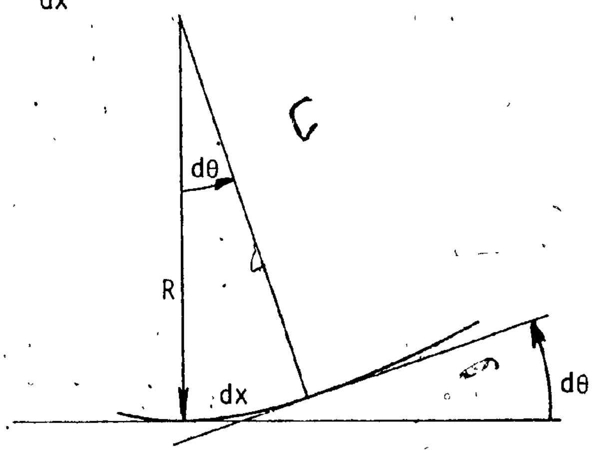


Figure 2.4.1

In addition to these relations we have, from the theory of beams, the flexure formula

$$\frac{1}{R} = \frac{M}{EI}$$

Where EI is the flexural rigidity of the beam and R is the radius of curvature.

Substituing for dO and 1/r,

$$U_{max} = \frac{1}{2} \int \frac{M^2}{EI} dx = \frac{1}{2} \int_0^{\ell} EI \left(\frac{d^2y}{dx^2}\right)^2 dx \tag{2.4.8}$$

Equating the kinetic and potential energies, the fundamental frequency of the beam is detremined from the equation

$$\omega^2 = \frac{\int_0^{\ell} EI \left(\frac{d^2y}{dx^2}\right)^2 dx}{\int_0^{\ell} my^2 \, dx} \tag{2.4.9}$$

Substituing W/g = m', eq. 2.4.9 becomes

$$\omega^2 = \frac{\int_0^{\ell} (EI) \left(\frac{d^2y}{dx^2}\right)^2 dx}{\int_0^{\ell} \frac{w}{g} y^2 \, dx} \qquad (2.4.10)$$

Example 2.3 An horizontal vessel is simply supported of uniform cross section, shown in fig. (2.4.2). Between vessel supports the distance is 65'-0" and the diameter is 36 in. Its thickness is ½" plate of material SA-516, Gr. 70. The operating weight if 64 000 lb. Find the fundamental natural frequency of the vessel?

SOLUTION

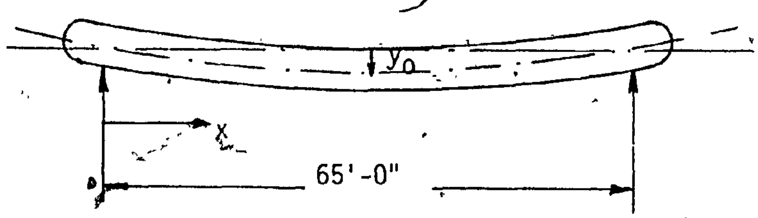


Figure 2.4.2

We assume the deflection to be represented by a sine curve as follows

$$y = (y_0 \sin \frac{\pi x}{\ell}) \sin \omega t$$

Where y_0 is the maximum deflection at mid-span. The second derivative then becomes

$$\frac{d^2y}{dx^2} = - \left(\frac{\pi}{\ell}\right)^2 y_0 \sin \frac{\pi x}{\ell} \sin \omega t$$

Substituing into eq. (2.4.10) we obtain

$$\omega^2 = \frac{EI\left(\frac{\pi}{\ell}\right)^4 \int_0^{\ell} sin^2 \frac{\pi x}{\ell}\, dx}{\frac{W}{g} \int_0^{\ell} sin^2 \frac{\pi x}{\ell}\, dx}$$

$$\omega^2 = \pi^4 \frac{gEI}{W\ell^4}$$

Where $E = 30\ 000\ 000$ *psi for SA-516, Gr. 70*

$$I = \pi R^3 h = \pi(36)^3\ (\tfrac{1}{2}) = 73287\ in^4$$

$$w = \frac{64000}{85 \times 12} = 63\ lb/in.$$

$$\ell = 65 \times 12 = 780\ in.$$

Natural frequency of the vessel ω_n

$$\omega_n = \sqrt{\frac{\pi^4 g\ EI}{w\,\ell^4}}$$

$$= \sqrt{\frac{386(30\ 000\ 000)\ (73\ 287)\pi^4}{63\ (780)^4}}$$

$$\omega_n = 60\ rad/sec.$$

Fundamental natural frequency

$$f_n = \frac{\omega_n}{2\pi} = \frac{60}{2\pi} = 9.5\ Hz$$

and the fundamental period T

$$T = \frac{1}{f_n} = \frac{1}{9.5} = 0.106\ sec.$$

Example 2.4 Consider the fractionator tower of example (2.2). By using Rayleigh's method, find the natural frequency of the tower?

SOLUTION

Assuming the following uniformed cantilever beam in fig. 2.4.3

$$w = \frac{64000}{73 \times 12} = 73\ lb/in$$

$$E = 30\ 000\ 000\ psi$$

$$\ell = 73'\text{-}0'' \times 12 = 876\ in.$$

$$I = 64\ 053\ in^4$$

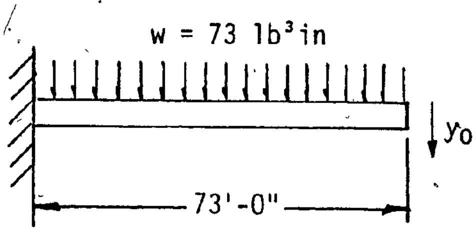


Figure 2.4.3

Also assuming here that the amplitude of the beam at any point x is given with suffisent accuracy by the statical deflection curve of a massless cantilever beam with a concentrated load at the end.

Writing this equation in the form

$$y = \frac{1}{2} y_o [3(\frac{x}{\ell})^2 - (\frac{x}{\ell})^3]$$

Where $y_o = \frac{P\ell^3}{3EI}$ *is the amplitude of the free end.*

$$\therefore k' = \frac{P}{y_o} = \frac{3EI}{\ell^3} \text{ is the stiffness at the free end.}$$

The potential energy which is equal to the work done, is then

$$U_{max} = \frac{1}{2} k' y_o^2 = \frac{3EI}{2\ell^3} y_o^2$$

This result can also be found from the equation

$$U_{max} = \frac{1}{2} \int_o^{\ell} EI \left(\frac{d^2y}{dx^2}\right)^2 dx$$

The kinetic energy is next determined by integrating

$$T_{max} = \frac{\omega^2}{2g} \int_o^{\ell} wy^2 \ dx$$

$$= \frac{\omega^2}{2g} \ w \left(\frac{y_o}{2}\right)^2 \int_o^{\ell} \left[3 \left(\frac{x}{\ell}\right)^2 - \left(\frac{x}{\ell}\right)^3\right]^2 dx$$

$$T_{max} = \frac{1}{2} \left(\frac{33 \cdot w\ell}{140 \ g}\right) \omega^2 y_o^2$$

The above equation indicates that for the assumed deflection curve, the continuous beam of w lb/ft is equivalent in vibration characteristics to that of a weightless beam with a concentrated weight $\left(\frac{33 \ w\ell}{140}\right)$ at the end.

Equating the two energies, the fundamental frequency of vibration in radians per second becomes

$$\omega_n = \sqrt{\frac{(3 \ EI/\ell^3)g}{\frac{33 \ w\ell}{140}}} = 3.56 \ \sqrt{gEI/w\ell^4}$$

$$\omega_n = 3.56 \sqrt{\frac{386(30 \cdot 000 \ 000) \ (64 \ 053)}{73 \ (876)^4}} = 14.79 \ \text{rad/sec.}$$

fundamental natural frequency f_n

$$f_n = \frac{\omega_n}{2\pi} = \frac{14.79}{2\pi} = 2.35 \ \text{cps.}$$

fundamental period T

$$T = \frac{1}{f_n} = \frac{1}{2.35} = 0.426 \ \text{sec.}$$

<u>*Note:*</u> *The exact solution for this case is given in example (2.2) by the equation*

$\omega_1 = 3.515 \sqrt{gEI/w\ell^4} = 14.60$ rad/sec.

$f_1 = 2.32$ cps

$T = 0.430$ sec.

In general, the deflection curve assumed for the problem should satisfy the boundary conditions of deflection, slope, shear, and moment. These conditions are satisfied by the static deflection curve which generally results in a frequency of acceptable accuracy.

2.4.2 Vibrating beams with lumped weights

Rayleigh's methods may also be applied to a beam represented by a series of lumped weights W_1, W_2, W_3 ... W_n, by assuming as before, a deflection of each mass according to the equation.

$$y_n = y_{on} \sin \omega_n t \tag{2.4.11}$$

The maximum deflections are therefore y_{01}, y_{02}, y_{03}, ... y_{on}, and the maximum velocities are $y_{01}\,\omega_n$, $y_{02}\,\omega_n$, $y_{03}\,\omega_n$, ... $y_{on}\,\omega_n$.

The maximum potential energy of the system is

$$U_{max} = 1/2\ W_1\ y_{01} + 1/2\ W_2\ y_{02} + 1/2\ W_3\ y_{03} + \ldots + 1/2\ W_n\ y_{on}$$

$$\therefore U_{max} = \frac{1}{2} \sum_{i=1}^{n} W_i\ y_{oi} \tag{2.4.12}$$

The maximum kinetic energy is

$$T_{max} = \frac{1}{2g} W_1\ y_{01}^2\ \omega_n^2 + \frac{1}{2g} W_2\ y_{02}^2\ \omega_n^2 + \frac{1}{2g} W_3\ y_{03}^2\ \omega_n^2 + \ldots + \frac{1}{2g} W_n\ y_{on}^2\ \omega_n^2$$

$$\therefore T_{max} = \frac{\omega_n^2}{2g} \sum_{i=1}^{n} W_i\ y_{oi}^2 \tag{2.4.13}$$

Applying Rayleigh's principle and solving for the natural frequency yield

$$\omega_n = \sqrt{\frac{g \ \Sigma W_i y_{oi}}{\Sigma W_i y_{oi}}} \qquad (2.4.14)$$

or $$f_n = \frac{1}{2\pi} \sqrt{\frac{g \ \Sigma W_i y_{oi}}{\Sigma W_i y_{oi}^2}} \qquad (2.4.15)$$

There are as many natural frequencies in a multimass vibrating system as there are degrees of freedom. Thus, a three-mass lateral system will have three degrees of freedom and, consequently, three natural frequencies. Equation 2.4.14 gives only the first or lowest of these frequencies.

The deflections of a beam, y_{01}, y_{02}, y_{03} ... y_{on}, due to concentrated load W_1, W_2, W_3 ... W_n, can be found from standard handbook deflection formulae. The deflections at the loads can be obtained using the principle of super-position of the effects of all loads [5].

2.4.3 Vessels with different wall thicknesses

For vessels with several different stiffness sections, Rayleigh's method can be used. This method lends itself to automatic computation using programmable calculators or computors.

For a cantilever beam, the deflection curve taken as the arbitraty function which must satisfy the boundary conditions of the system is

$$y = c \ x^2 \ (6\ell^2 - 4\ell x + x^2) \qquad (2.4.16)$$

The kinetic and potential energies for this case are

$$T_{max} = \frac{1}{4}\omega^2 \int_0^{\ell} my^2 \, dx \qquad (2.4.17)$$

$$U_{max} = \frac{1}{4} E \int_0^{\ell} I \left(\frac{d^2y}{dx^2}\right)^2 dx \qquad (2.4.18)$$

Equating eq. (2.4.17) and (2.4.18)

The natural frequency is

$$\omega^2 = E \frac{\int_0^{\ell} I \cdot \left(\frac{d^2y}{dx^2}\right)^2 dx}{\int_0^{\ell} my^2 \, dx} \qquad (2.4.19)$$

Substituing $m = \frac{W}{g}$ and splitting the integrals into sections with constant load and constant moment of inertia,

$$\omega^2 = Eg \frac{I \int_0^{\ell} \left(\frac{d^2y}{dx^2}\right)^2 dx + (I_1 - I) \int_0^{\ell_1} \left(\frac{d^2y}{dx^2}\right)^2 dx + \ldots}{W \int_0^{\ell} y^2 \, dx + (W_1 - W) \int_0^{\ell_1} y^2 \, dx + \ldots} \qquad (2.4.20)$$

Substituing (2.4.16) and differentiating as indicated

$$\int_0^{\ell_i} \left(\frac{d^2y}{dx^2}\right)^2 dx = 144\, C^2 \int_0^{\ell_i} (\ell - x)^4 \, dx = -\frac{144}{5} C^2 (\ell - \ell_i)^5 \qquad (2.4.21)$$

$$\int_0^{\ell_i} y^2 \, dx = C^2 \int_0^{\ell_i} x^4 (6\ell^2 - 4\ell x + x^2)^2 \, dx$$

$$= C^2 \ell_i^5 \left(7.2\ell^4 - 8\ell^3\ell_i + 4\ell^2\ell_i^2 - \ell\ell_i^3 + \frac{1}{9}\ell_i^4\right) \qquad (2.4.22)$$

Expressions are obtained which can easily be programmed for a calculator with limited storage capacity because parameters of each section of different stiffness can be expressed in terms of the parameters of the adjacent section [6].

2.5 Method of influence coefficients [4], [7]

This method is also used to find the natural frequency of a system of lumped masses. The equation of motion for such a system in matrix form is

$$[m]\,[\ddot{y}] = -\,[K]\,[y] \tag{2.5.1}$$

2.5.1 can first be premultipled by $[K^{-1}]$, and with the assumption of harmonic motion $\ddot{y} = -\lambda y$, where $\lambda = \omega^2$, we obtain the equation

$$[A^{-1} - \lambda^{-1} I][y] = 0 \tag{2.5.2}$$

The matrix $[A^{-1}] = [K^{-1}][m]$ is the flexibility matrix $[a]$

$$\therefore [A^{-1}] = [K^{-1}][m] = [a] \tag{2.5.3}$$

Equation 2.5.2 is an abbreviated form of the equations of motion formulated on the basis of the flexibility influence coefficients. The flexibility influence coefficient a_{ij} is defined as the displacement at i due to a unit force applied at j. With forces f_1, f_2 and f_3 acting at stations 1, 2, and 3, the principle of superposition can be applied to determine the displacements in termes of the flexibility influence coefficients.

$$\begin{aligned} y_1 &= a_{11}f_1 + a_{12}f_2 + a_{13}f_3 \\ y_2 &= a_{21}f_1 + a_{22}f_2 + a_{23}f_3 \\ y_3 &= a_{31}f_1 + a_{32}f_2 + a_{33}f_3 \end{aligned} \tag{2.5.4}$$

and $[a]$ the flexibility matrix is

$$[a] = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \tag{2.5.5}$$

Assuming harmonic motion and replacing the forces f_i by the inertia forces

$$-m_i\ddot{y}_i = \omega^2 m_i y_i \tag{2.5.6}$$

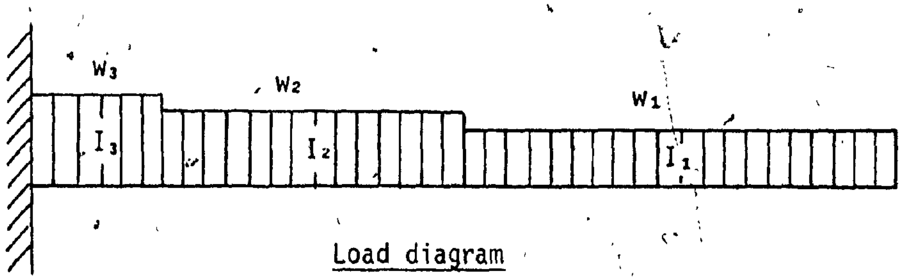
Eq. 2.5.4 then becomes

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \omega^2 \begin{bmatrix} a_{11}m_1 & a_{12}m_2 & a_{13}m_3 \\ a_{21}m_1 & a_{22}m_2 & a_{23}m_3 \\ a_{31}m_1 & a_{32}m_2 & a_{33}m_3 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \tag{2.5.7}$$

This matrix can be evaluated for ω^2 using an iteration procedure. The iteration procedure is started by assuming a set of deflections for the right column of eq. 2.5.7 and performing the indicated operation, which results in a column of numbers. This is then normalized by making one of the amplitudes equal to unity and dividing each term of the column by the particular amplitude which was normalized. The procedure is then repeated with the normalized column until the amplitudes stabilize to a definite pattern.

The iteration process converges to the lowest value of ω^2, the fundamental frequency or the lowest mode of vibration.

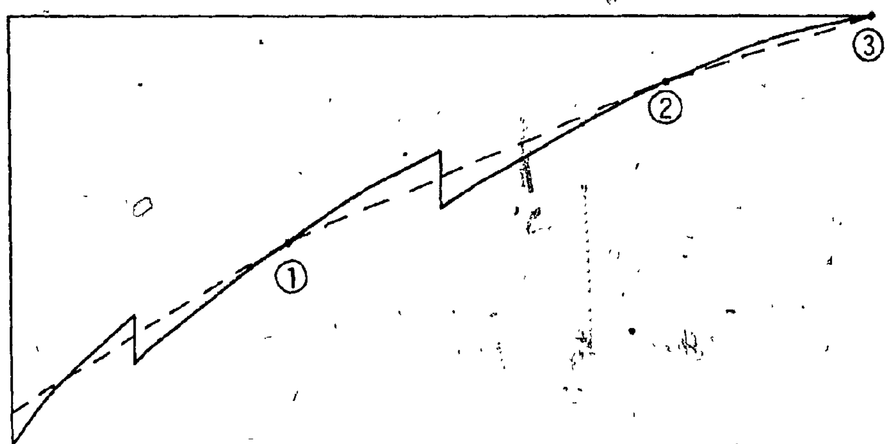
In order to apply this method of influence coefficients to vertical pressure vessels, we must find the deflection coefficient of the system thought to be in horizontal position, cantilevered from its base and loaded by its own weight, fig. (2.5.1).



Load diagram

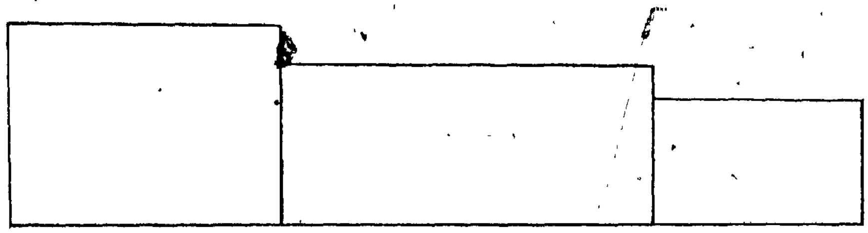
Figure 2.5.1

Secondly the M/I diagram fig. (2.5.2) is constructed assuming a constant modulus of elasticity E.

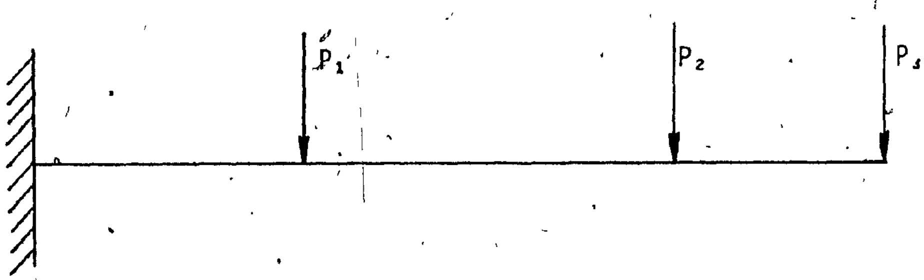


M/I diagram
Figure 2.5.2

Then finally by finding the equivalent shear diagram, fig. (2.5.3) and the concentrated loads at point, 1, 2, and 3, fig. (2.5.4) produces the equivalent load diagram



Equivalent shear diagram
Figure 2.5.3



Equivalent load diagram
Figure 2.5.4

Example 2.5 Find the natural frequency and the fundamental period of the fractionator tower 54" dia. and 140 ft. high for the plate thicknesses and loading conditions as shown below

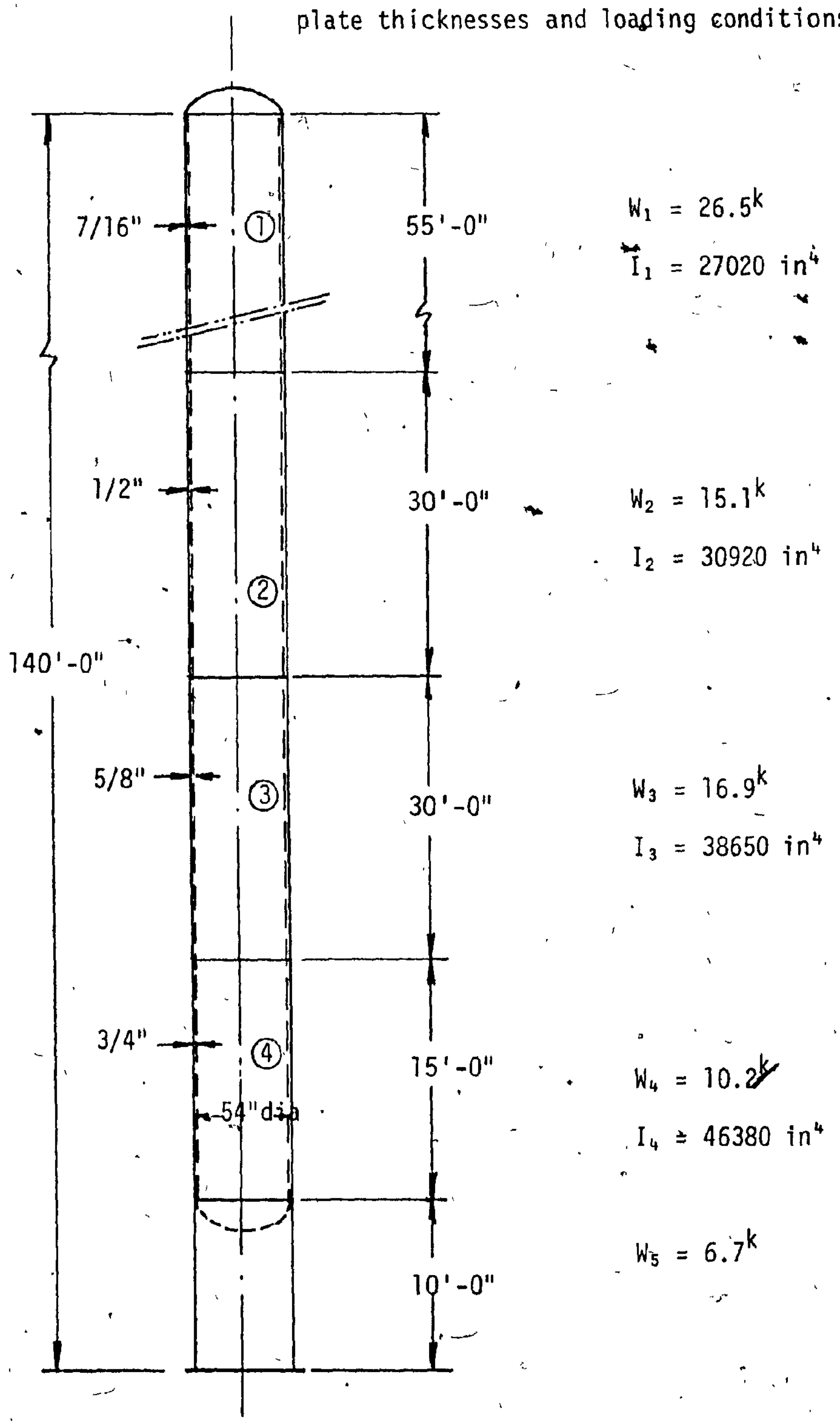


Figure 2.5.5

SOLUTION

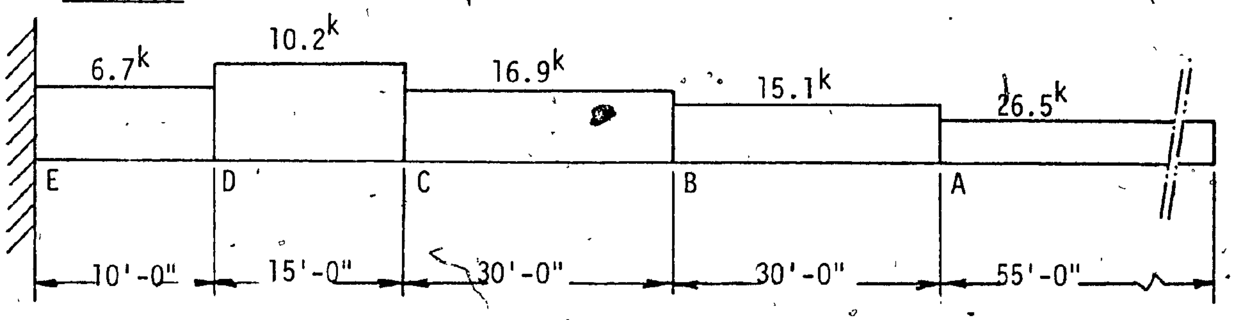


Figure 2.5.6

Bending moment at point A

$$M_A = (26.5)\left(\frac{55}{2}\right) = 729 \text{ ft - k}$$

Bending moment at point B

$$M_B = (26.5)\left(\frac{55}{2} + 30\right) + (15.1)\left(\frac{30}{2}\right) = 1750 \text{ ft - k}$$

Bending moment at point C

$$M_C = (26.5)\left(\frac{55}{2} + 30 + 30\right) + (15.1)\left(\frac{30}{2} + 30\right) + (16.9)\left(\frac{30}{2}\right) = 3252 \text{ ft - k}$$

Bending moment at point D

$$M_D = (26.5)\left(\frac{55}{2} + 30 + 30 + 15\right) + (15.1)\left(\frac{30}{2} + 30 + 15\right) + (16.9)$$
$$\left(\frac{30}{2} + 15\right) + (10.2)\left(\frac{15}{2}\right) = 4206 \text{ ft - k}$$

Bending moment at point E

$$M_E = (26.5)\left(\frac{55}{2} + 30 + 30 + 15 + 10\right) + (15.1)\left(\frac{30}{2} + 30 + 15 + 10\right)$$
$$+ (16.9)\left(\frac{30}{2} + 15 + 10\right) + (10.2)\left(\frac{15}{2} + 10\right) + (6.7)\left(\frac{10}{2}\right) = 4926 \text{ ft - k}$$

The bending moments are then multiplied by $\frac{I_1}{I_i}$ *as follows,*

$$\frac{I_1}{I_2} M_A = \frac{27020}{30920}(729) = 637 \text{ ft - k}$$

$$\frac{I_1}{I_2} M_B = \frac{27020}{30920} (1750) = 1530 \text{ ft - k}$$

$$\frac{I_1}{I_3} M_B = \frac{27020}{38650} (1750) = 1223 \text{ ft - k}$$

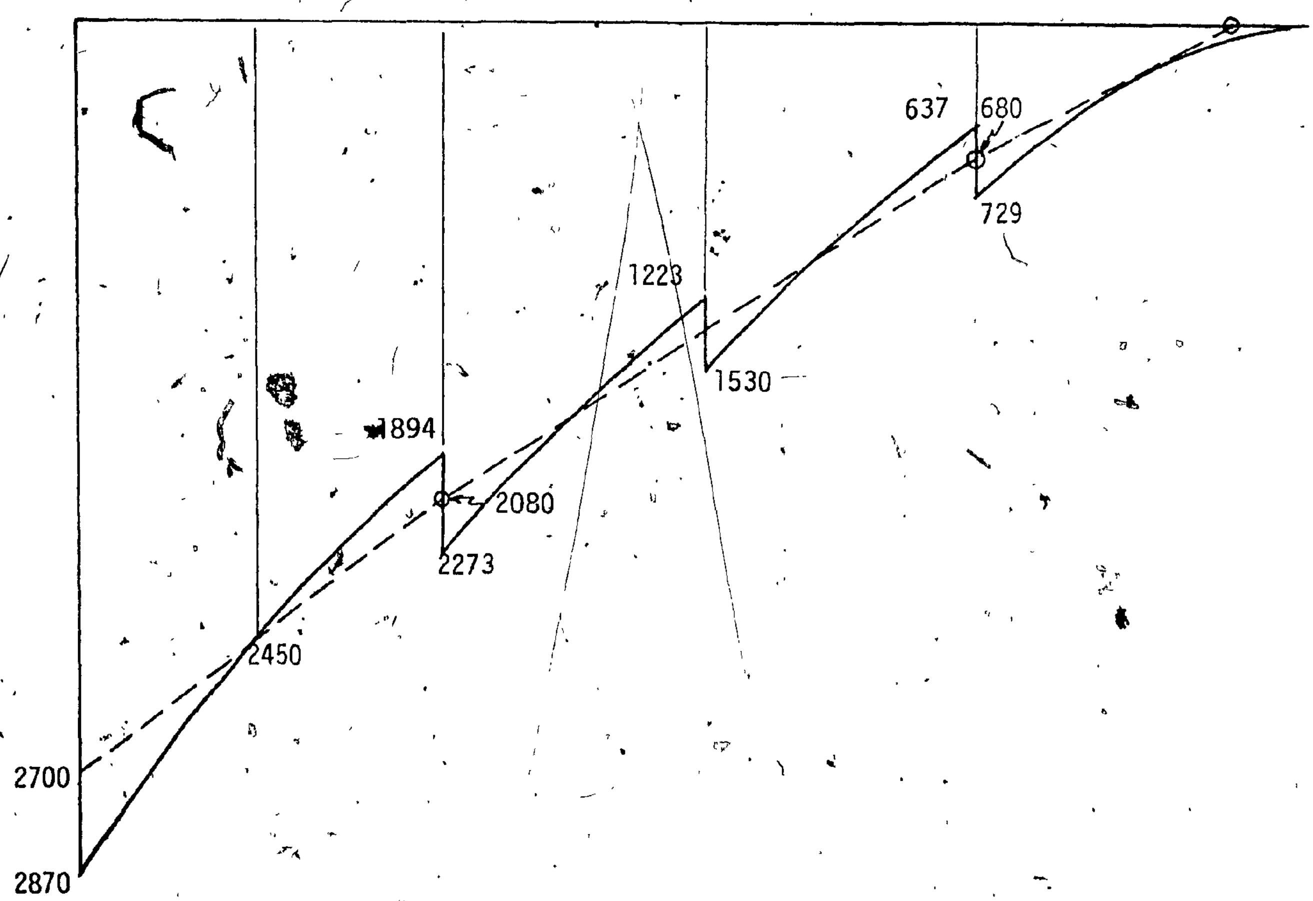
$$\frac{I_1}{I_3} M_C = \frac{27020}{38650} (3252) = 2273 \text{ ft - k}$$

$$\frac{I_1}{I_4} M_C = \frac{27020}{26380} (3252) = 1894 \text{ ft - k}$$

$$\frac{I_1}{I_4} M_D = \frac{27020}{46380} (4206) = 2450 \text{ ft - k}$$

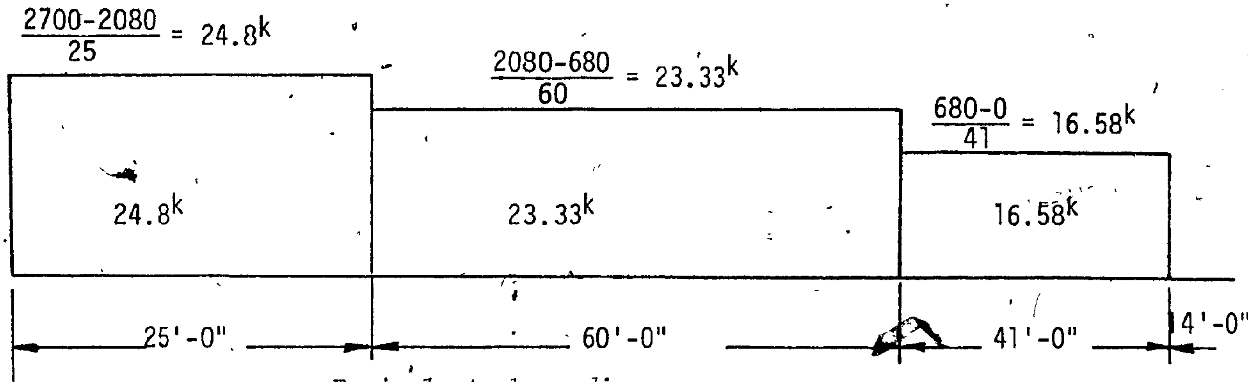
$$\frac{I_1}{I_4} M_E = \frac{27020}{46380} (4926) = 2870 \text{ ft - k}$$

and plotted, see fig. (2.5.7)



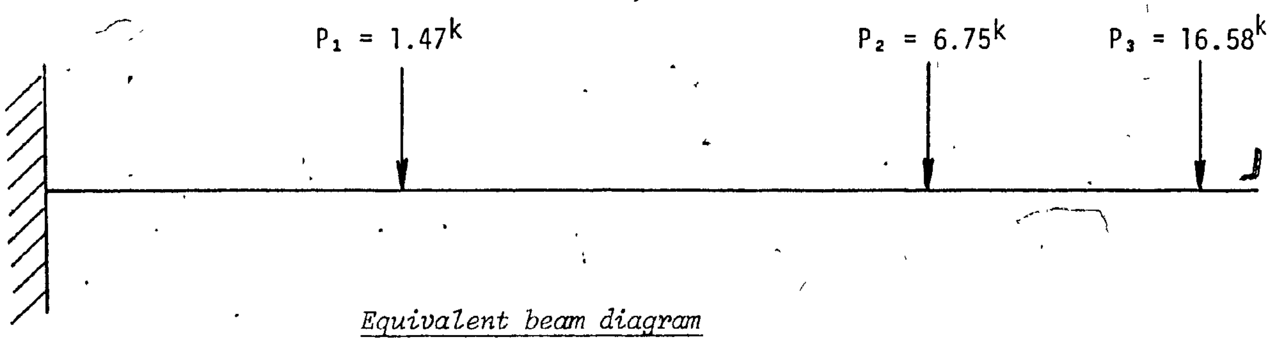
Bending moment diagram $\frac{M}{I}$ (I_1)

Figure 2.5.7



Equivalent shear diagram

Figure 2.5.8



Equivalent beam diagram

Figure 2.5.9

The deflection coefficients a_{ij}, using the equivalent system, are determined from handbook deflection formulas

$$a_{ij} = \frac{\ell_i^2}{6EI}(3\ell_j - \ell_j) = a_{ji}$$

$$\therefore\ a_{11} = \frac{25^2}{6EI}[3(25) - 25](12^3) = \frac{2(25^3)(12^3)}{6EI} = \frac{(9.0)(10^6)}{EI}$$

$$a_{22} = \frac{85^2}{6EI}[3(85) - 85](12^3) = \frac{2(85^2)(12^3)}{6EI} = \frac{(353.7)(10^6)}{EI}$$

$$a_{33} = \frac{126^2}{6EI}[3(126) - 126](12^3) = \frac{2(126^2)(12^3)}{6EI} = \frac{(114330)(10^6)}{EI}$$

$$a_{12} = a_{21} = \frac{25^2}{6EI}[3(85) - 25](12^3) = \frac{(41.4)(10^6)}{EI}$$

$$a_{13} = a_{31} = \frac{25^2}{6EI}[3(126) - 25](12^3) = \frac{(63.54)(10^6)}{EI}$$

$$a_{23} = a_{32} = \frac{85^2}{6EI}[3(126) - 85](12^3) = \frac{(609.67)(10^6)}{EI}$$

$$\therefore \quad [y_i] = \frac{\omega^2}{gEI}[a_{ij}\ P_i][y_i]$$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \frac{(10^{12})\omega^2}{g(EI)} \begin{bmatrix} 0.0132 & 0.2794 & 1.0535 \\ 0.0608 & 2.3874 & 10.1083 \\ 0.0934 & 4.1153 & 18.9509 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

To find $[y_i]$ *we assume a vector, say* $[1, 4, 8]$ *and multiply it with* $[a_{ij}\ P_i]$

$$\begin{bmatrix} 0.0132 & 0.2794 & 1.0535 \\ 0.0608 & 2.3874 & 10.1083 \\ 0.0934 & 4.1153 & 18.9509 \end{bmatrix} \begin{bmatrix} 1 \\ 4 \\ 8 \end{bmatrix} = \begin{bmatrix} 9.5588 \\ 90.4768 \\ 168.1618 \end{bmatrix} = 9.5588 \begin{bmatrix} 1.000 \\ 9.4653 \\ 17.5924 \end{bmatrix}$$

Repeating the procedure with the new vector $\begin{bmatrix} 1.0000 \\ 9.4653 \\ 17.5924 \end{bmatrix}$ *leads to*

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \frac{(10^{12})\omega^2}{gEI} \begin{bmatrix} 21.1914 \\ 200.4875 \\ 372.4378 \end{bmatrix} = \frac{(21.1914)\ (10^{12})\omega^2}{gEI} \begin{bmatrix} 1.0000 \\ 9.4608 \\ 17.5749 \end{bmatrix}$$

$$= \frac{(21.1914)\ (10^{12})\omega^2}{gEI} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix}$$

$$\therefore \quad \frac{(21.1914)\ (10^{12})\omega^2}{gEI} = 1$$

$$\omega = \sqrt{\frac{(386)\ (29)\ (10^6)\ (27020)}{(21.1914)\ (10^{12})}} = 3.78 \text{ rad/sec}$$

$$f_n = \frac{\omega}{2\pi} = \frac{3.78}{2\pi} = \underline{\underline{0.60 \text{ Hz}}}$$

$$T = \frac{1}{f_n} = \frac{1}{0.60} = \underline{\underline{1.66 \text{ sec.}}}$$

2.6 Natural frequencies of cylindrical shells

LEISSA's [14] [15] work contains a comprehensive presentation of available results for free vibration frequencies and mode shapes of cylinders, in a format which can be used directly by a design engineer, whenever natural frequencies of pressure vessels are required for seismic analyses.

LEISSA compares the results of various shell theories, with those from exact, three dimensional elastic analysis. In addition to the differences between the various theories, simplifications are often made in the resulting equations of motion or the characteristic (frequency) equations. These simplifications include, among others: neglecting certain small terms in the equation of motion, neglect of the tangential inertia terms, linearization of the characteristic equations, and assuming that the wave length in one direction is considerably longer than in the other.

The mode of the fundamental or lowest frequency of a cylindrical shell of finite length is generally not at all obvious, since the complete frequency spectrum is determined independent variation of four (4) parameters:

The number of circumferential waves,
the thickness-to-radius ratio,
the length-to-radius ratio,
Poisson's ratio.

Shell vibrations are more complicated than plate vibrations, because bending stresses of a shell cannot, in general, be separated from membrane stresses. Thus, the classical bending theory of shells is governed by an eighth order system of governing partial differential equations of motion, while the corresponding plate bending equation is only of the fourth order. Also the boundary conditions for shells required by the classical bending theory are four, instead of two for plates. To demonstrate the significance of the latter point, consider a flat plate which is simply supported along two of its opposite edges. The number of possible problems which can then arise, considering all combinations of "simple" boundary conditions which can exist on the remaining two edges, is 10. For a cylindrically shell the corresponding number is 136. We cannot obviously describe all these combinations in this section of this report, but we can cover the main typical combinations that are found in practical application such as:

- shear diaphragm (SD-SD)
- clamped-free

Consider the closed circular cylindrical shell of finite length ℓ, with circular cover plate at each end, see fig. (2.6.1). The plates would have considerable stiffnesses in their own planes, thereby restraining the v and w components of shell displacement at their mutual boundaries. However, the plates, by virtue of their thinness, would have very little stiffness in the x and u direction transverse to their planes; consequently, they would generate a negligible bending moment M_x and longitudinal membrane force N_x in the shell as the shell deforms.

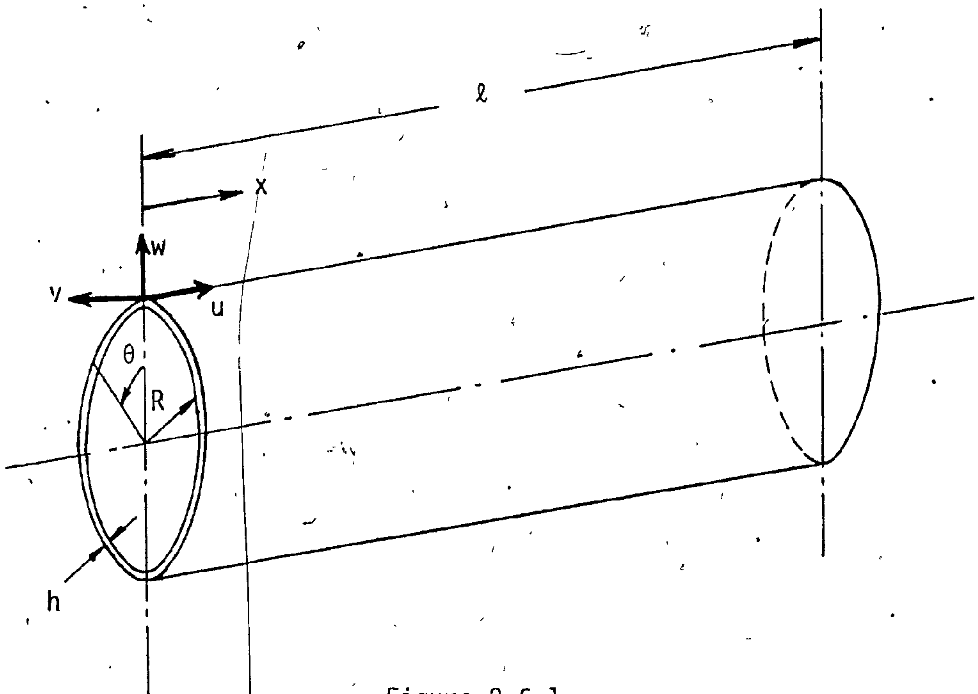


Figure 2.6.1

Consider the closed circular cylindrical shell of finite length having displacements of the form

$$u = A \cos \lambda s \cos n\theta \cos \omega t$$
$$v = B \sin \lambda s \sin n\theta \cos \omega t \qquad (2.6.1)$$
$$w = C \sin \lambda s \cos n\theta \cos \omega t$$

Where A, B, C, and λ are undetermined constants, n is an integer for closed shells (see fig. 2.6.2), and ω is the frequency of free vibration in radians per second when the mass density ρ is expressed in units involving seconds. By taking one of the set of homogenous equations, (from the Donnell-Mushtari theory, for example), the equations of motion can be written in matrix form as shown in equation (2.6.2).

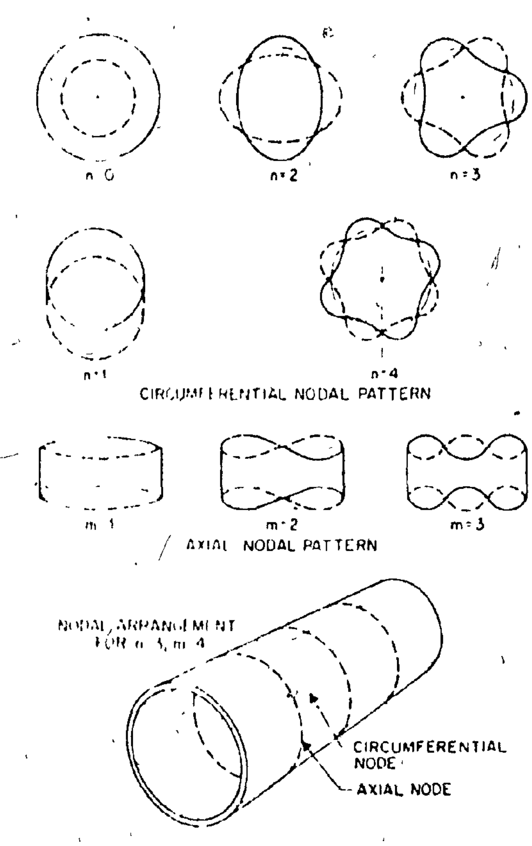


FIGURE 2.6.2 Nodal patterns for circular cylindrical shells supported at both ends by shear diaphragms. (After ref (4)

$$\begin{bmatrix} [-\lambda^2 - \frac{(1-\nu)}{2} n^2 + \frac{\rho(1-\nu^2)R^2\omega^2}{E}] & \frac{(1+\nu)}{2}\lambda n & \nu\lambda \\ \frac{(1+\nu)}{2}\lambda n & [-\frac{(1-\nu)}{2}\lambda^2 - n^2 + \frac{\rho(1-\nu^2)R^2\omega^2}{E}] & -n \\ -\nu\lambda & n & [1 + k(\lambda^2+n^2) - \frac{\rho(1-\nu^2)R^2\omega^2}{E}] \end{bmatrix} \begin{bmatrix} A \\ B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \qquad (2.6.2)$$

Where $k = h^2/12R^2$

ν = Poisson's ratio

For a nontrivial solution, the detreminant of the coefficient matrix in equation (2.6.2) is set equal to zero, which yields either of the following two eigenvalue problems:

(1) For a given λ, there exists one or more proper values of the frequency parameter Ω such that the detreminant vanishes,

$$\Omega^2 = \frac{\rho(1-\nu^2)R^2\omega^2}{E} \qquad (2.6.3)$$

(2) For a given frequency ω, there exists one or more proper values of λ such that the detreminant vanishes. Of course, since $\xi = x/R$, then the half-wavelength of the displacement functions in the x direction is ℓ if $\lambda = \frac{\pi R}{\ell}$ and the frequencies of free vibration can be found which correspond to the given wavelength.

One simple mathematical model of a cylindrical shell is obtained by using the concept of "plane strain". The necessary assumptions are that there is no motion in the direction of the length of the shell and that the physical quantities (displacements, membranes forces, bending moments, etc.) do not depend upon location along the length. Thus, the case of place strain requires

$$u = o, \quad v = v(\theta), \quad w = w(\theta) \qquad (2.6.4)$$

which changes the character of the shell motion from two-dimentional to one-dimensional (variation only with θ) and simplifies the analysis considerably.

By using this simplified analysis with the assumption of equation (2.6.4) and applying the Flügge shell theory, the following equations of motion can be written in matrix form as such:

$$\begin{bmatrix} n^2 - \Omega^2 & n \\ n & 1 + k(1 - n^2)^2 - \Omega^2 \end{bmatrix} \begin{bmatrix} B \\ C \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \qquad (2.6.5)$$

where Ω^2 is same as in equation (2.6.3) and $k = h^2/12R^2$
For a nontrivial solution, setting the determinant of the coefficient matrix in equation (2.6.5) equal to zero gives the roots

$$\left.\begin{aligned} &\Omega^2 = 0,\ 1 + k && \text{for } (n = 0) \\ &\Omega^2 = \frac{1}{2}[1 + n^2 + k(n^2 - 1) \\ &\quad \pm \sqrt{[1 + n^2 + k(n^2 - 1)^2]^2 - 4kn^2(n^2-1)^2]} \\ & && \text{for } (n \neq 0) \end{aligned}\right\} \qquad (2.6.6)$$

Note that the root $\Omega^2 = 0$ for $n = 0$ corresponds to rigid body torsional rotation of the shell.

There has been a certain number of results found using the Flügge shell theory, and the frequencies and the fundamental frequency (i.e. the lowest frequency for all n) could be found easily by using graphs in which the frequency parameter $\Omega = \omega R \quad \rho(1 - \nu^2)/E$ is plotted as a function of the length/radius ratio ℓ/mR for various numbers of circumferential waves n.

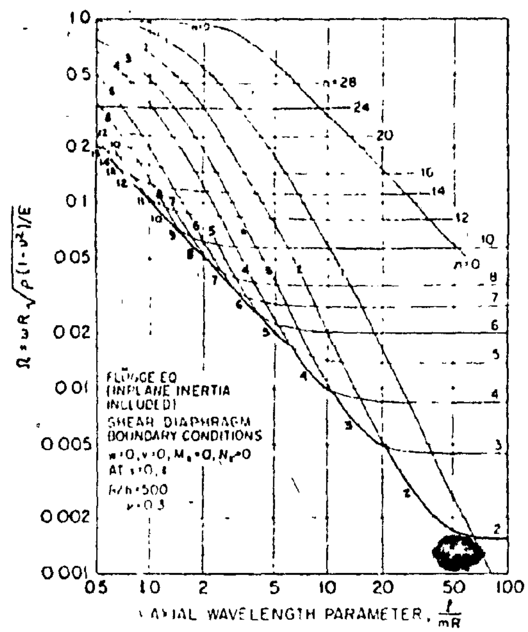


FIGURE 2.6.3 -Variation of the frequency parameter Ω according to the Flugge theory ($R/h = 500$). (After ref. 14)

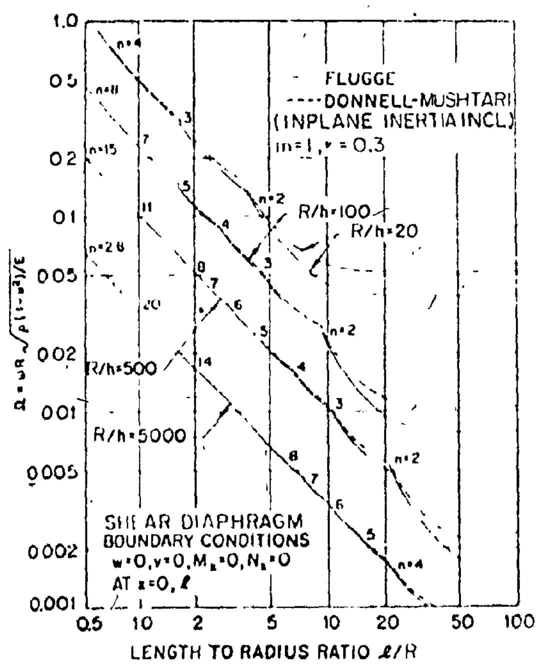


FIGURE 2.6.4 Fundamental frequency parameters Ω for various l/R and R/h ratios. (After ref. 14)

Following are some results presented for closed thin circular cylindrical shells of finite lenght, for the simple boundary conditions mentioned before.

2.6.1 Shear diaphragms at both end

An other terminology for SHEAR DIAPHRAGMS (SD-SD) is when a shell is "simply-supported".

The circular cylindrical shell, shown in fig. (2.6.1), supported at both ends by shear diaphragms, has received by far the most attention in the litterature. This is due to the fact that one simple form of the solutions to the eighth order differential equations of motion is also capable of satisfying the SD-SD boundary conditions exactly.

These boundaries conditions are:

$$w = M_x = N_x = V = 0 \quad \text{at} \quad x = 0, \ell \tag{2.6.7}$$

These conditions can be closely approximated in physical application simply by means of rigidly attaching a thin, flat, circular cover plate at each end. The plates would have considerable stiffnesses in their own planes, thereby restraining the v and w components of shell displacement at their mutual boundaries. However, the plates, by virtue of their thinness, would have very little stiffness in the x direction transverse to their planes; consequently, they would generate negligible bending moment M_x and longitudinal membrane force N_x in the shell as the shell deforms. Because of the capability of the plates to supply shearing forces $N_{x\theta}$ to the shell, the type of boundary conditions satisfied by equation (2.6.7) is called SHEAR DIAPHRAGM.

Flügge, by using his theory arrived at the following results presented in fig. (2.6.3) to fig. (2.6.13).

It is abvious from figure 2.6.3 that, for a fixed number of circumferential waves, the frequency increases with an increased number of longitudinal half-waves m, and that the fundamental (lowest) frequency always occurs for m = 1, but for varying n depending strongly upon the length/radius ratio of the shell.

The fundamental frequencies, which are given by the envelope of figure (2.6.3) when m = 1, are shown in figure (2.6.4) for various R/h ratios.

Figures (2.6.5) through (2.6.8) show lowest values of Ω plotted versus ℓ/mR for R/h = 20, 50, 100 and 2000 respectively. In these figures the solid lines correspond to motions which are primarily radial (A < C, B < C). However, it is also seen that for the axisymmetric (n = 0) and beam bending (n = 1) modes, as ℓ/mR is increased, the motions became axial and mixed, respectively, as shown by the dashed lines. In figure 2.6.5 the envelope of the frequency curves establishes the fundamental frequency for the R/h ratio of 20. It is interesting to note that for shells having an ℓ/R ratio in the vicinity of unity, the fundamental frequency is associated with four circumferential waves (n = 4), whereas for both larger and smaller ℓ/R ratios the fundamental frequency occurs for smaller n. For very short shells ($\ell/R < 0.3$) it is seen that the fundamental mode is axisymmetric.

Figures 2.6.9 and 2.6.10 show the frequency spectra of the second and third eigenvalues for given n and λ. A single figure covers the range of R/h from 20 to 5000 for modes corresponding to the second and third eigenvalues. For small values of ℓ/mR and n the second eigenvalue yields

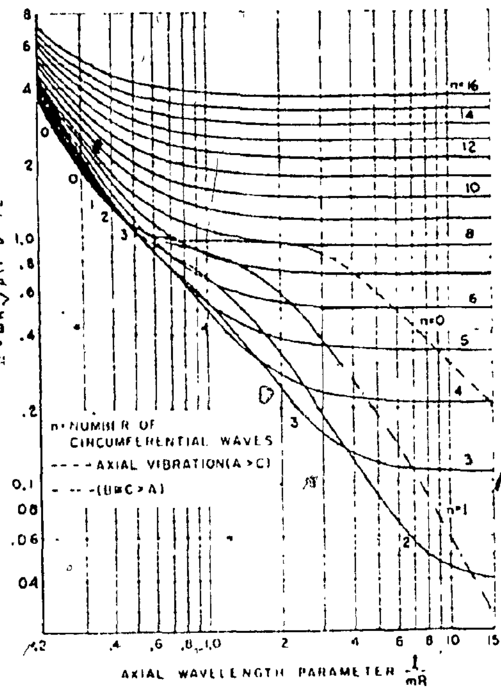


FIGURE 2.6.5 -Variation of the fundamental frequency parameter Ω with l/mR according to the Flugge theory, $\nu = 0.3$, $R/h = 20$. (After ref. 14)

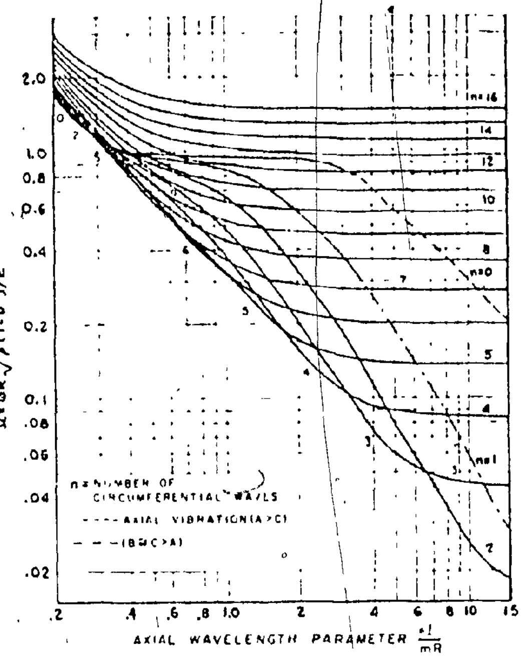


FIGURE 2.6.6 -Variation of the fundamental frequency parameter Ω with l/mR according to the Flugge theory; $\nu = 0.3$, $R/h = 50$. (After ref. 14)

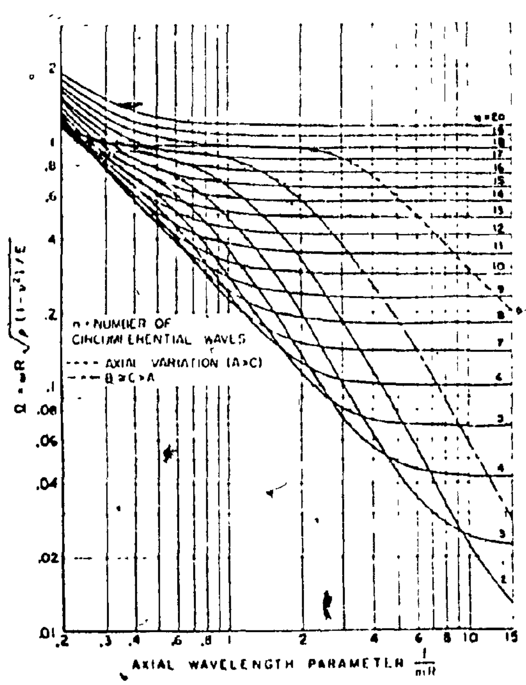


FIGURE 2.6.7 -Variation of the fundamental frequency parameter Ω with l/mR according to the Flugge theory; $\nu = 0.3$, $R/h = 100$. (After ref. 14)

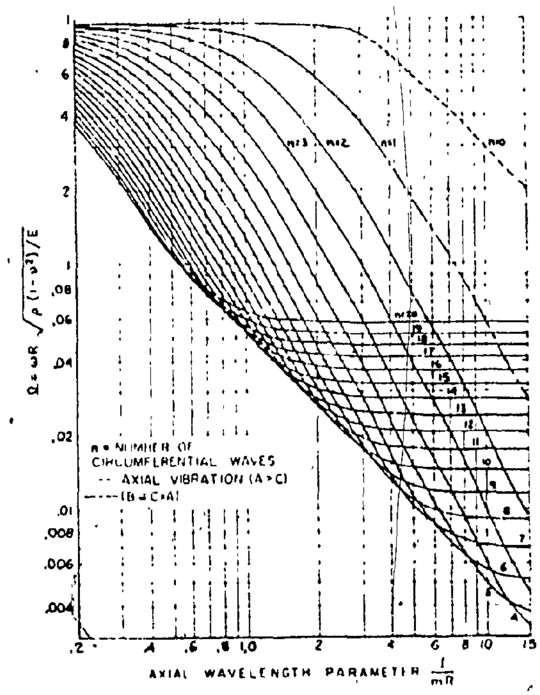


FIGURE 2.6.8 -Variation of the fundamental frequency parameter Ω with l/mR according to the Flugge theory; $\nu = 0.3$, $R/h = 2000$. (After ref. 14)

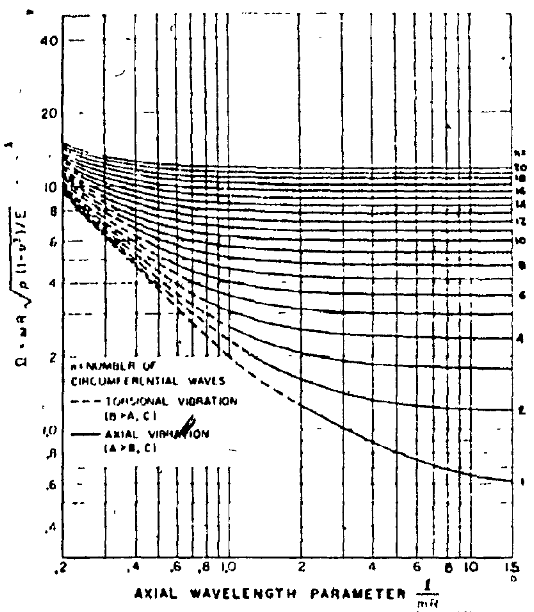


FIGURE 2.6.9 -Second vibration frequencies; Flugge theory, $\nu=0.3$, $R/h=20$ to 5000. (After ref. 14)

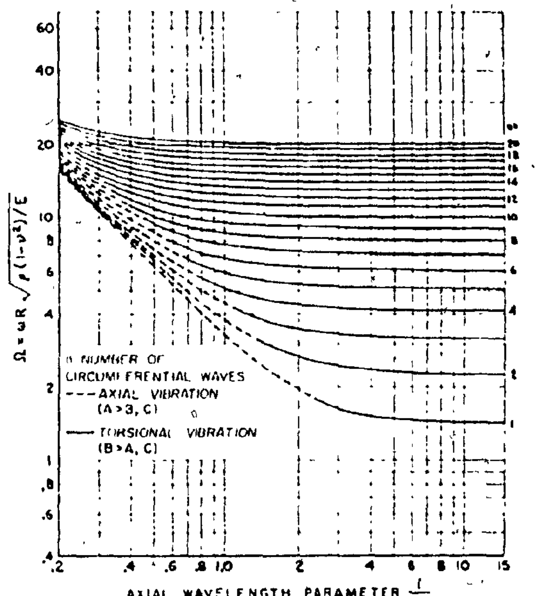


FIGURE 2.6.10-Third vibration frequencies; Flugge theory, $\nu=0.3$, $R/h=20$ to 5000. (After ref. 14)

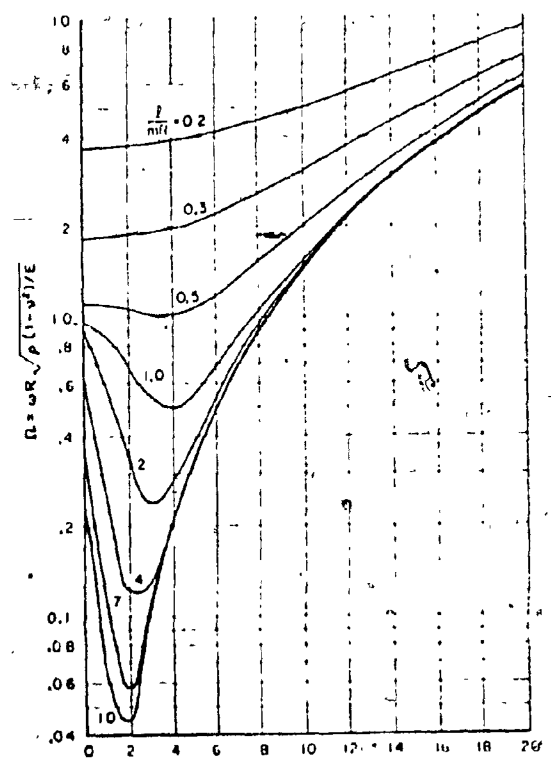


FIGURE 2.6.11—Variation of the fundamental frequency parameter Ω with n; Flugge theory, $\nu=0.3$, $R/h=20$. (After ref. 14)

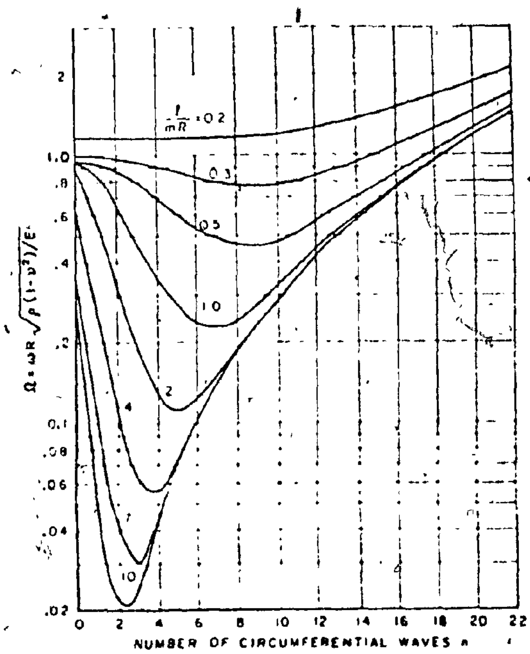


FIGURE 2.6.12 -Variation of the fundamental frequency parameter Ω with n; Flugge theory, $\nu=0.3$, $R/h=100$. (After ref. 14)

amplitude ratios such that B > A, C (torsional modes) while modes having larger ℓ/mR and n have amplitude ratios such that A > B, C (axial modes). The converse of this is found for the third eigenvalues.

In figures 2.6.11, 2.6.12 and 2.6.13, the lowest frequency parameter is plotted versus n for R/h = 20, 100, and 2000 respectively. This last set of figures serves to emphasize clearly that the minimum frequency for a thin circular cylindrical shell of given length and supported by shear diaphragms occurs for n = 2 or greater, unless $\ell/R > 10$. On the other hand, for very long shells, the minimum frequency always occurs for n = 1, that is in the beam bending mode.

The values of the frequency parameter Ω arising from the three-dimensional elasticity solutions are given in table 2.6.1 and could be compared with the results given by the figure 2.6.11.

Another interesting set of frequency spectra is shown in figures 2.6.14 through 2.6.17. In these figures, the frequency Ω is plotted versus λ giving rise to a family of curves for different thickness ratios in the range $0.002 < h/R < 0.100$. Looking at these curves, it is obvious that the frequency increases as the length of the shell decreases and as h/R increases; but, in addition, as one moves from figure 2.6.14 to figure 2.6.17, it is apparent that the family of curves spreads apart, indicating greater frequency differences with increasing h/R for larger values of n.

2.6.2 Clamped-free and other boundary conditions

Another terminology for CLAMPED-FREE is when a shell or a vertical vessel is mounted with a CANTILEVER BEAM effect. The circular cylindrical

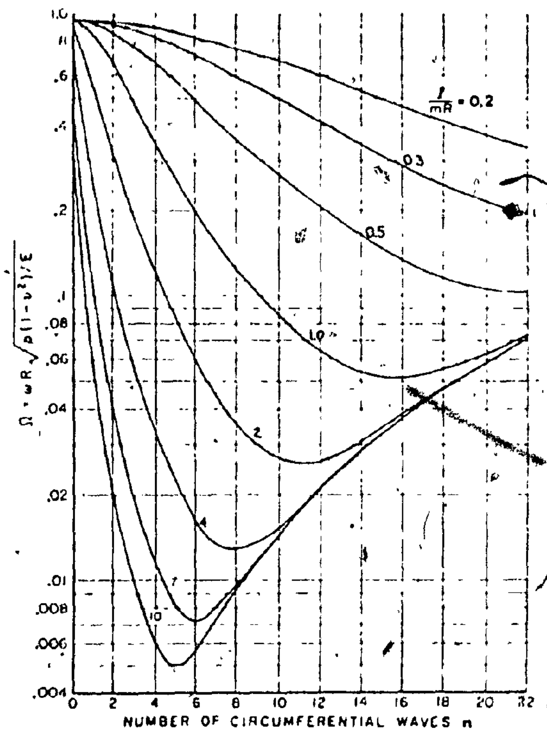


FIGURE 2.6.13 -Variation of the fundamental frequency parameter Ω with n; Flugge theory, $\nu = 0.3$, $R/h = 2000$ (After ref. 14)

TABLE 2.6.1 -*Lowest Frequency Parameters According to Three-Dimensional Theory; SD-SD' Supports;* $\nu = 0.3$

R/h	n	l/mR					
		0.1	0.25	1	4	20	100
20	0	10.4586	2.28505	0 958083	0.464618	0 0929296	0 0185859
	1	10.4670	2.29380	.856114	257011	0164063	.000665031
	2	10.4914	2.32044	.675486	.121249	0392332	.0347711
	3	10.5326	2.36597	539294	.129881	109477	.109186
	4	10.5898	2.43231	.492343	219098	209008	.208711
500	0	(1.11103)	(.957994)	.949203	.464648	0929296	.0185859
	1	(1.11019)	(.951993)	.844952	.256883	.0161011	.00266482[illegible]
	2	(1.10890)	(.934462)	.652148	112689	.00515213	.00156235
	3	(1.10630)	(.906734)	.481028	0580087	.00503721	.00138626
	4	(1.10270)	(.870765)	.354118	0353927	.00853109	.00840299

Note: Values in parentheses are from the Flugge shell theory.

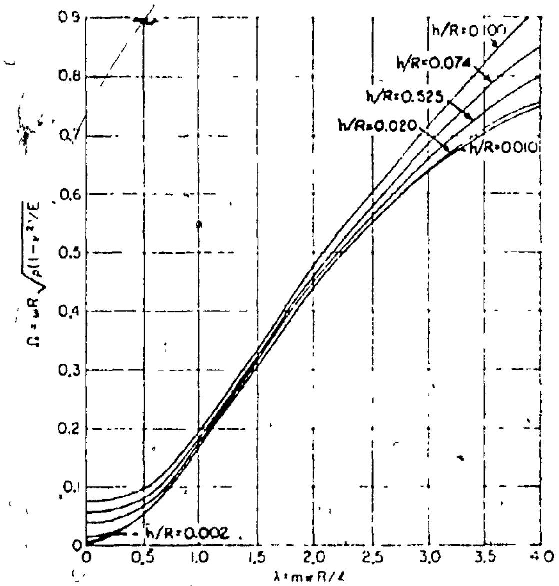


FIGURE 2.6.14-Variation of the fundamental Ω with λ and h/R; Arnold and Warburton theory, $n=2$ (After ref. 14)

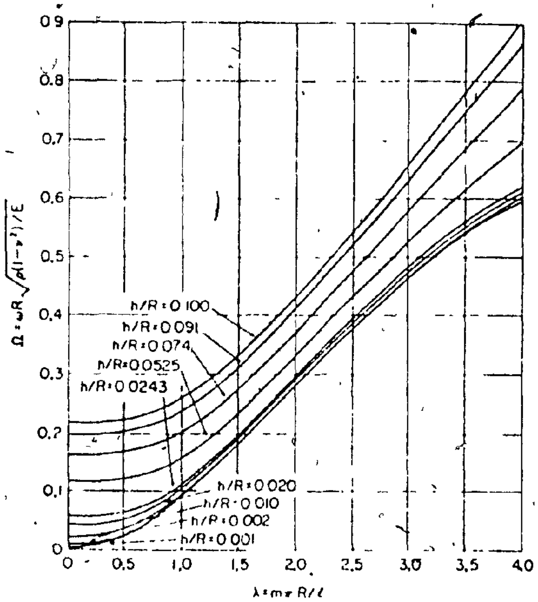


FIGURE 2.6.15-Variation of the fundamental Ω with λ and h/R, Arnold and Warburton theory, $n=3$. (After ref. 14)

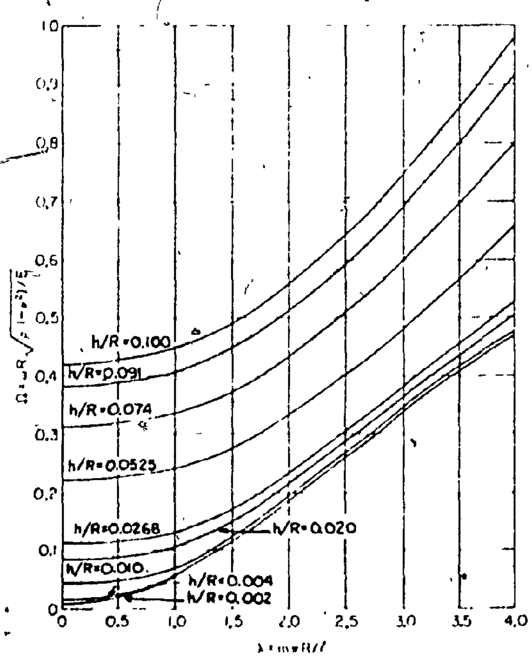


FIGURE 2.6.16-Variation of the fundamental Ω with λ and h/R; Arnold and Warburton theory, $n=4$. (After ref. 14)

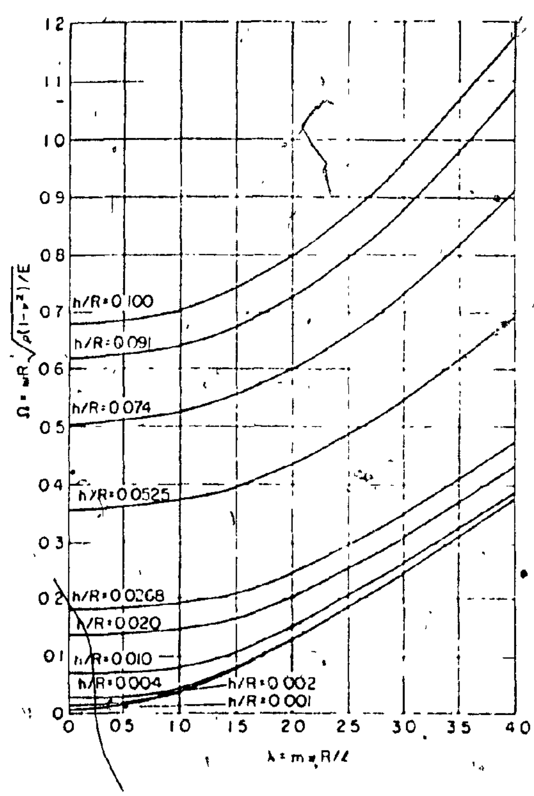


FIGURE 2.6.17 Variation of the fundamental Ω with λ and h/R; Arnold and Warburton theory, $n=5$. (After ref. 14)

shell, shown in fig. (2.6.1), is clamped or fixed at one end and free at the other. The boundary conditions for this case are:

$$\left.\begin{aligned} & u = v = w = \frac{\partial w}{\partial x} = 0 \quad \text{at } x = 0 \\ & N_x = N_{x\theta} + \frac{M_{x\theta}}{R} = Q_x + \frac{1}{R}\frac{\partial M_{x\theta}}{\partial \theta} = M_x = 0 \quad \text{at } x = \ell \end{aligned}\right\} \qquad (2.6.8)$$

By assuming solution functions which are generalizations of equation (2.6.1), it is possible to obtain exact solutions for the frequencies of free vibration for this case, although the amount of computational work required is relatively great. Flügge suggested a procedure in order to find the exact solution for the frequencies and mode shapes of free-vibration for each of the remaining 135 cases.

Lowest frequency parameters are given by Gontkevich who uses the Rayleigh-Ritz method with beam functions to obtain characteristic equations for the six boundary conditions having clamped, shear diaphragm, or free end conditions at either or both ends of a circular cylindrical shell.

The Rayleigh-Ritz method using beam functions and the Donnell-Mushtari shell theory are the basis for the results shown in figures (2.6.18) through (2.6.21) for the case of clamped-free condition. Admissible values of ε_m for the abscissa of figures (2.6.18) through (2.6.21) are available in table (2.6.2) for some of the main cases of boundary conditions. Mode shapes for the lowest frequencies for $m = 1$ and $m = 2$ are shown in figure (2.6.22). Numerical results were obtained for the clamped-free case using the Rayleigh-Ritz method in conjunction with the Flügge shell equations. Displacement functions were assumed in the form

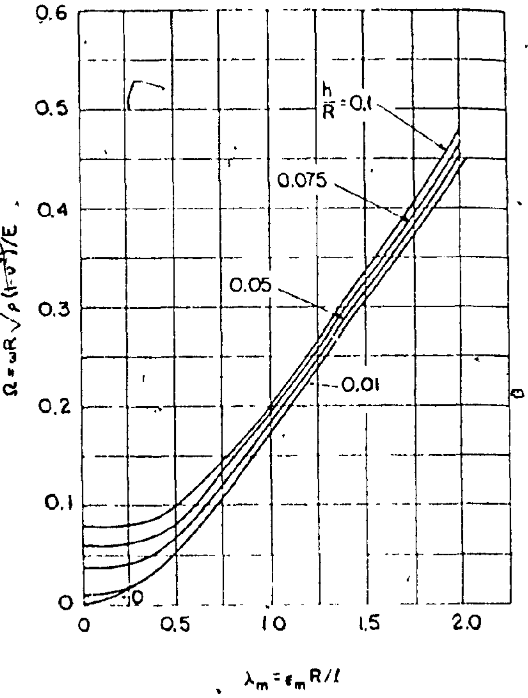


FIGURE 2.6.18 Lowest frequency parameters for clamped-free shells (see table 2.6.2 for admissible ϵ_m), $n = 2$. (After ref. 14)

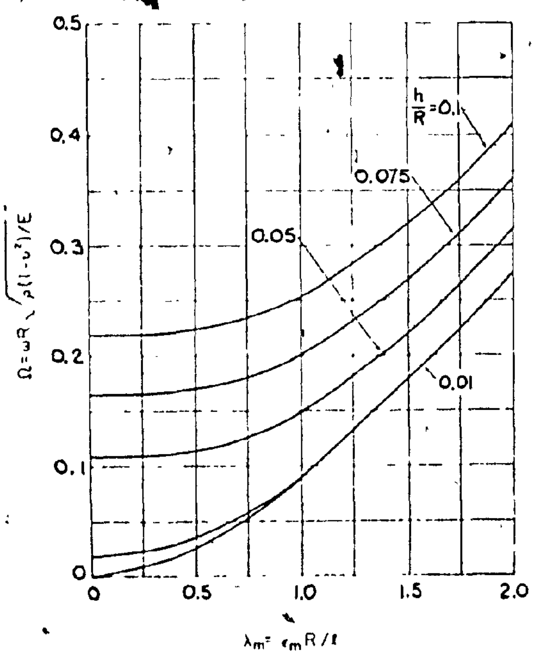


FIGURE 2.6.19 Lowest frequency parameters for clamped-free shells (see table 2.6.2 for admissible ϵ_m); $n = 3$. (After ref. 14)

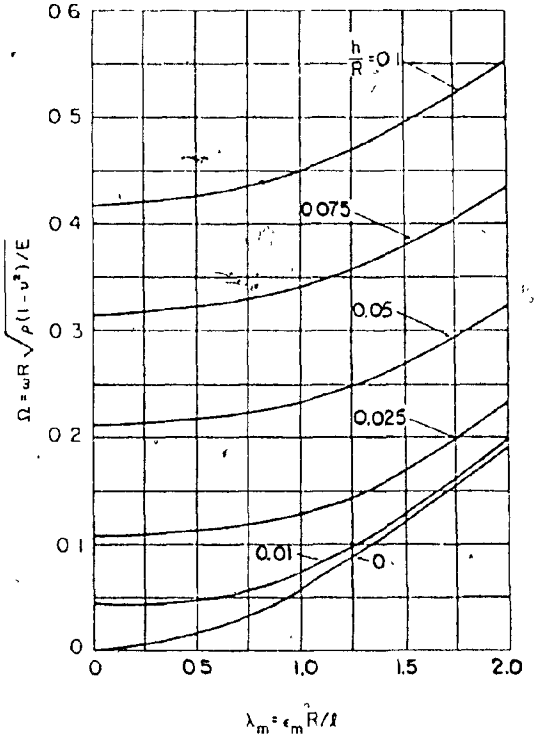


FIGURE 2.6.20-Lowest frequency parameters for clamped-free shells (see table 2.6.2 for admissible ϵ_m); $n = 1$. (After ref. 14)

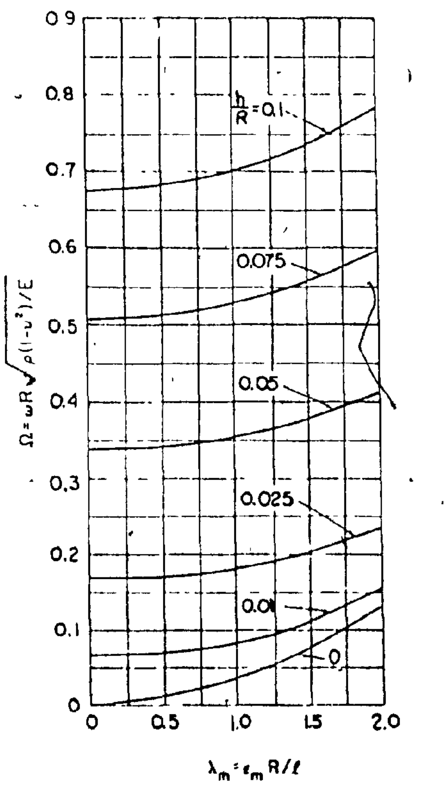


FIGURE 2.6.21 -Lowest frequency parameters for clamped-free shells (see table 2.6.2 for [illegible]

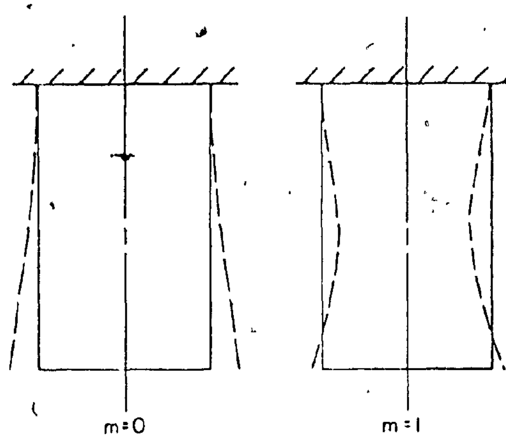


FIGURE 2.6.22.–Mode shape for a clamped-free shell.

TABLE 2.6.2 –*Constants for the Characteristic Equation*

m	Item	SD-SD	Clamped-clamped	Clamped-free	Free-free	Clamped-SD	SD-free
0		—	—	1.321886		—	—
1		1.0	0.549880	1.471208	2.211601	0.723422	1.742905
2		1.0	.746084	1.252875	1.766169	.856926	1.422809
3		1.0	.818051	1.181963	1.515592	.902022	1.293787
4	δ_m	1.0	.858553	1.141465	1.421119	.925136	1.224729
5		1.0	.884249	1.115749	1.317244	.939525	1.181899
>5		1.0	$1-\frac{2}{\left(m+\frac{1}{2}\right)\pi}$	$1+\frac{2}{\left(m+\frac{1}{2}\right)\pi}$	$1+\frac{6}{\left(m+\frac{1}{2}\right)\pi}$	$1-\frac{1}{\left(m+\frac{1}{4}\right)\pi}$	$1+\frac{3}{\left(m+\frac{1}{4}\right)\pi}$
0				0.244094			—
1				−.603337	−0.519879		−0.723122
2				−.714024	−.714024		−.902022
3				−.818169	−.818051		−.902022
4	γ_m	$-\delta_m$	$-\delta_m$	−.858524	−.858533	$-\delta_m$	−.925136
5				−.869100	.884249		−.939525
>5				$-1+\frac{2}{\left(m+\frac{1}{2}\right)\pi}$	$-1+\frac{2}{\left(m+\frac{1}{2}\right)\pi}$		$-1+\frac{1}{\left(m+\frac{1}{4}\right)\pi}$
0		—		1.875104		—	
1		π	4.73004	4.69409	4.73004	3.92660	3.92660
2		2π	7.853204	7.854757	7.853204	7.06858	7.06858
3		3π	10.995608	10.995541	10.995608	10.2102	10.2102
4	ϵ_m	4π	14.137166	14.137168	14.137166	13.3518	13.3518
5		5π	17.27876	17.27880	17.27876	16.4934	16.4934
>5		$m\pi$	$\frac{(2m+1)}{2}\pi$	$\frac{(2m+1)}{2}\pi$	$\frac{(2m+1)}{2}\pi$	$\frac{(4m+1)}{4}\pi$	$\frac{(4m+1)}{4}\pi$

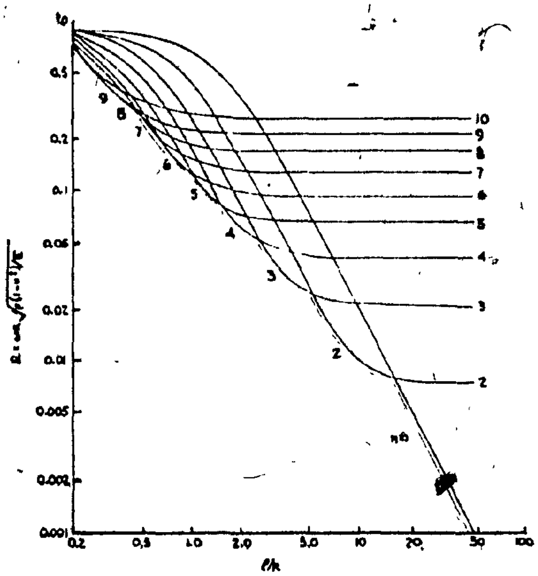


FIGURE 2.6.23 Frequency parameters for clamped-free shells, $m = 1$, $\nu = 0.3$, $R/h = 100$. (After ref. 14)

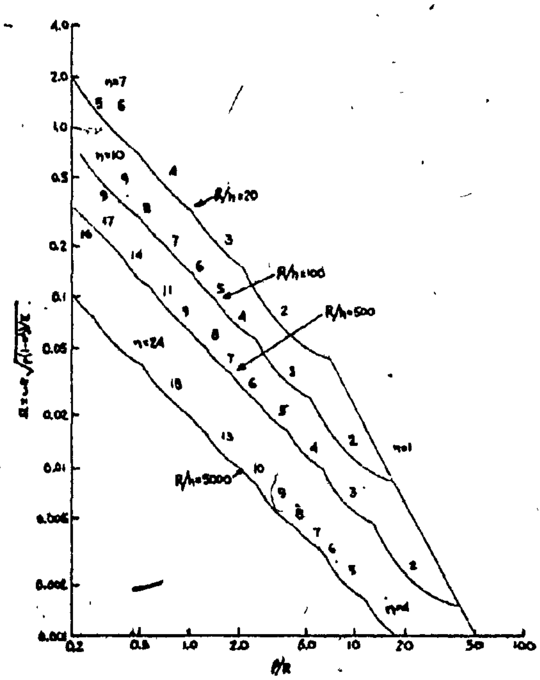


FIGURE 2.6.24-Frequency envelopes for clamped-free shells; $m = 1$, $\nu = 0.3$. (After ref. 14)

TABLE 26.3 *Frequency Parameters* $\omega R\sqrt{\rho(1-\nu^2)/E}\times 10^2$ *for a Clamped-Free Shell;* $m=1$, $\nu=0.3$

L/R	Degree of char. eq.	R/h = 50, n = 1	50, n = 2	100, n = 1	100, n = 2	150, n = 1	150, n = 2	200, n = 1	200, n = 2	250, n = 1	250, n = 2	300, n = 1	300, n = 2
10	S	2.0835	1.7084	2.0834	1.0351	2.0834	0.8510	2.0834	0.7808	2.0834	0.7445	2.0834	0.7240
	C	2.2012	1.7226	2.2011	1.0619	2.2011	.8871	2.2010	.8171	2.2010	.7826	2.2010	.7632
	Q	2.4579	1.7528	2.4578	1.1094	2.4578	.9132	2.4578	.8776	2.4578	.8155	2.4578	.8276
15	S	.9106	1.5867	.9105	.8351	.9105	.6001	.9105	.4921	.9105	.4341	.9105	.3973
	C	.9981	1.5892	.9983	.8415	.9983	.6096	.9983	.5038	.9983	.4464	.9983	.4119
	Q	1.0994	1.5958	1.0993	.8533	1.0993	.6256	1.0993	.5230	1.0993	.4679	1.0993	.4351
20	S	.5420	1.5632	.5320	.7953	.5320	.5150	.5320	.4238	.5320	.3539	.5320	.3095
	C	.5655	1.5638	.5651	.7973	.5651	.5182	.5654	.4281	.5651	.3594	.5621	.3151
	Q	.6198	1.5660	.6198	.8013	.6198	.5539	.6198	.4353	.6198	.3677	.6198	.3251
25	S	.3414	1.5560	.3414	.7838	.3414	.5288	.3414	.4030	.3414	.3290	.3414	.2807
	C	.3631	1.5561	.3630	.7845	.3630	.5300	.3630	.4048	.3630	.3311	.3630	.2832
	Q	.3971	1.5570	.3971	.7864	.3971	.5324	.3971	.4079	.3971	.3349	.3971	.2876
30	S	.2374	1.5534	.2374	.7794	.2374	.5227	.2374	.3952	.2374	.3191	.2374	.2695
	C	.2526	1.5534	.2526	.7797	.2526	.5232	.2526	.3960	.2526	.3204	.2526	.2706
	Q	.2759	1.5535	.2759	.7805	.2750	.5244	.2759	.3975	.2759	.3223	.2759	.2720
35	S	.1746	1.5517	.1746	.7774	.1746	.5200	.1746	.3918	.1746	.3152	.1746	.2645
	C	.1858	1.5517	.1858	.7775	.1858	.5202	.1858	.3924	.1858	.3156	.1858	.2650
	Q	.2028	1.5519	.2028	.7780	.2028	.5209	.2028	.3929	.2028	.3167	.2028	.2663
40	S	.1339	1.5509	.1339	.7764	.1339	.5186	.1339	.3900	.1339	.3131	.1339	.2620
	C	.1423	1.5509	.1423	.7764	.1423	.5187	.1423	.3902	.1423	.3133	.1423	.2623
	Q	.1553	1.5510	.1553	.7767	.1553	.5191	.1553	.3907	.1553	.3139	.1553	.2630
45	S	.1061	1.5504	.1061	.7758	.1061	.5179	.1061	.3891	.1061	.3120	.1061	.2607
	C	.1125	1.5504	.1125	.7758	.1125	.5179	.1125	.3891	.1125	.3120	.1125	.2608
	Q	.1227	1.5505	.1227	.7760	.1227	.5181	.1227	.3895	.1227	.3121	.1227	.2612
50	S	.0863	1.5501	.0862	.7755	.0862	.5174	.0862	.3885	.0862	.3113	.0862	.2599
	C	.0912	1.5501	.0912	.7755	.0912	.5174	.0912	.3885	.0912	.3113	.0912	.2599
	Q	.0991	1.5501	.0994	.7756	.0991	.5176	.0994	.3887	.0991	.3116	.0994	.2602
∞	S,C,Q	0	1.5192	0	.7746	0	.5164	0	.3873	0	.3098	0	.2582

Notes:

(1) S = sextic.
(2) C = cubic.
(3) Q = quadratic.

$$\left.\begin{aligned} u &= A_1\psi\,(x)\cos n\theta\cos wt \\ v &= B_1\psi\,(x)\sin n\theta\cos wt \\ w &= C_1\psi\,(x)\cos n\theta\cos wt \end{aligned}\right\} \qquad (2.6.9)$$

where $\psi(x)$ is the clamped-free beam function. This equation leads to a determinant of the third degree.

Frequency curves obtained using this third degree determinant are shown in figure (2.6.23) for $m = 1$, $\nu = 0.3$ and $R/h = 100$. Envelopes for various R/h ratios are given in figure (2.6.24). Numerical results obtained using third degree (cubic) frequency equations are listed in table (2.6.3) for the swaying ($n = 1$) and ovalling ($n = 2$) modes of long shells, such as tall pressure vessels and smokestacks.

Example 2.6

A horizontal pressure vessel as shown below is supported by two saddles. The inside diameter of the vessel is 36" and the tangent to tangent length is 13'-6". The overall thickness is ¼" stainless steel SA-240-304L material and the vessel operates at 250°F. Find the fundamental frequency and period of oscillation of the cylinder.

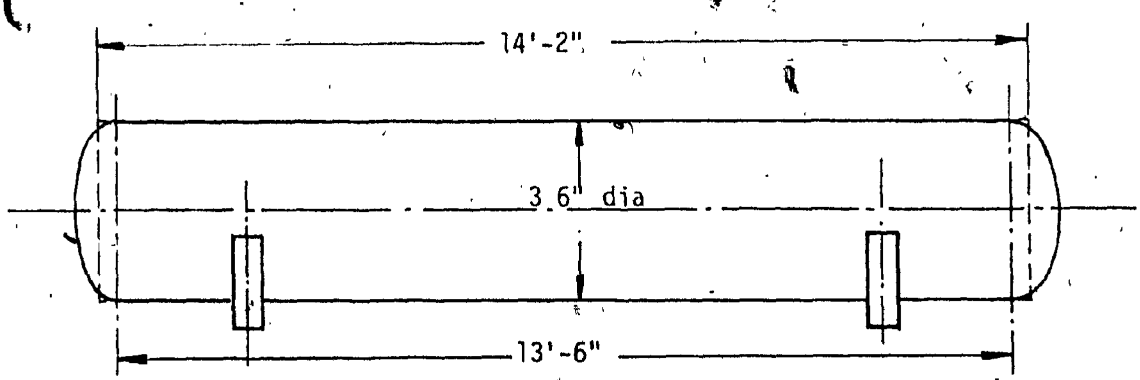


Figure 2.6.25

SOLUTION

Actual shape of vessel approximated by cylinder with flat ends.

$$\frac{R}{h} = \frac{18''}{.250} = 72 \,, \qquad \frac{\ell}{mR} = \frac{170}{(1)(18)} = 9.44$$

From fig. 2.6.12, it can be seen that the fundamental frequency occurs when n = 2 (2 radial nodes)

$$\lambda = \frac{m\pi R}{\ell} = \frac{(1)\pi(18)}{170} = .33$$

$$\frac{h}{R} = \frac{.250}{18} = .014$$

From fig. 2.6.14 (n = 2) for $\lambda = .33$ *and* $\frac{h}{R} = 0.014$

$$\Omega = \omega R \quad (1 - \nu^2)/E = 0.04$$

Fundamental frequency of the cylinder is

$$\omega = \sqrt{\frac{(386)(.04^2)(27.4)(10^6)}{(.283)(1-0.3^2)(18^2)}} = 450 \text{ rad/sec}$$

$$f_n = \frac{\omega}{2} = \frac{450}{2} = 72 \text{ cps}$$

Fundamental period

$$T = \frac{1}{f_n} = \frac{1}{72} = 0.014 \text{ sec.}$$

<u>Example 2.7</u>

A vertical pressure-vessel as shown below is supported by 3 legs. The inside diameter is 24" and the height between tangent lines is 31". The top head is tori-spherical and the bottom head conical. The overall thickness is ¼" stainless steel SA-240-304L material and the vessel operates at 150°F. Find the fundamental frequency and period of oscillation of the cylinder.

<u>*SOLUTION*</u>

The actual shape has been approximated by a cylinder with flat ends.

$$\frac{R}{h} = \frac{12}{.250} = 48$$

$$\frac{\ell}{mR} = \frac{38}{(1)(12)} = 3.17$$

From fig. 2.6.12 it can be seen that the fundamental frequency occurs when $m = 1$ $n = 4$ i.e. (1 axial, 4 radial nodes)

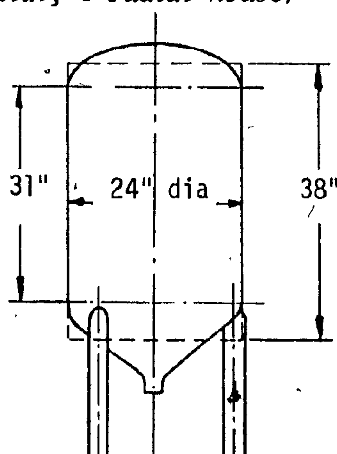


Figure 2.6.26

For clamped-free condition from table 2.6.2

for $m = 1$, $\varepsilon_m = 4.69409$

$$\therefore \lambda_m = \varepsilon_m R/\ell = 4.69409 \frac{(12)}{18} \doteq 1.48$$

$$\frac{h}{R} = \frac{.250}{12} = 0.020$$

From fig. 2.6.20 ($n = 4$) for $\lambda_m = 1.48$ and $\frac{h}{R} = 0.020$

$$\Omega = \omega R \sqrt{\rho(1 - \nu^2)/E} = 0.15$$

$$\omega = \sqrt{\frac{\Omega^2 E}{\rho(1-\nu^2)R^2}}$$

Fundamental frequency of the cylinder is

$$\omega = \sqrt{\frac{(386)(0.15^2)(28)(10^6)}{(.283)(1-.3^2)(12^2)}} = 2560 \text{ rad/sec}$$

$$f_n = \frac{\omega}{2\pi} = \frac{2560}{2\pi} = 407 \text{ cps}$$

Fundamental period T

$$T = \frac{1}{f_n} = \frac{1}{407} = 0.0024 \text{ sec}$$

3.0 SEISMIC ANALYSIS OF NUCLEAR POWER PLANT EQUIPMENT (A.E.C.L.)

3.1 Purpose

The specifications for seismic analysis [8] of nuclear power plant equipment by Atomic Energy of Canada Limited (A.E.C.L.) cover the minimum requirements of seismic analysis acceptable to AECL for equipment purchased and installed under their jurisdiction. Two methods are listed, their choice should be based on the practicallity of the method chosen for the type, size, shape and complexity of the equipment in question.

3.2 Seismic specifications

3.2.1 General requirements

The equipment shall be seismically qualified to the requirements of category "A" or category "B" and to the Design Basis Earthquake (DBE) level or to the Site Design Earthquake (SDE) level.

a) Category "A"

Equipment designated as category "A" shall be capable of maintaining its pressure retaining capability and structural integrity during and following the earthquake.

b) Category "B"

Equipment designated as category "B" shall be capable of maintaining its functional operability, which is the capability of performing the function for which it was designed, including pressure retaining capability and structural integrity during and following the earthquake.

c) Design Basis Earthquake (DBE)

The Design Basis Earthquake is a hypothetical earthquake. Its effects at the equipment location are defined by the DBE response spectra in two horizontal directions and one vertical direction, or in two horizontal directions with a definition of the response in the vertical direction.

d) Site Design Earthquake (SDE)

The Site Design Earthquake is a scaled down version of the Design Basis Earthquake. The Site Design Earthquake level is applied to specific equipment which is not required to survive a DBE, but which must survive an earthquake of lesser intensity to minimize economic loss or to maintain capability to perform a safety function following such an earthquake.

When the earthquake level is unknown, the equipment shall be qualified for the Design Basis Earthquake. The intensity of the Site Design Earthquake shall be defined as one half of the values shown for each frequency on the floor response spectra for the Design Basis Earthquake. The qualification shall combine the worst effects of seismic response in each of the two specified horizontal directions and the vertical direction.

The floor response spectra for each equipment support point shall be made available to the designer.

e) Floor Response Spectra

A Floor Response Spectra is a plot of the maximum response, in terms of displacement, velocity or acceleration, of a group of single degree of freedom oscillators of different natural frequencies rigidly mounted on the specified floor when the floor is subjected to the earthquake motion. The response is expressed as a function of the natural frequency or period of the oscillators, and plots are shown for selected percentages of critical damping of the oscillators. Each floor response spectra shows the response of the oscillators for a specified horizontal or vertical direction.

f) Natural frequency

The frequency or frequencies at which a body vibrates due to its own physical characteristics and elastic restoring forces brought into play when the body is distorted in a specific direction and then releazed, while restrained or supported at specified points.

g) Rigid Equipment

Rigid equipment is that for which the lowest natural frequency is 33 Hz and greater. Rigid equipment is generally that for which the lowest natural frequency is greater than twice the highest significant natural frequency of the supporting structure.

h) Damping

Damping is a measure of the energy dissipation in a vibrating body due to friction, impact, hysteresis, joint slippage, and changes caused by equipment loading which affect structural stiffness, support characteristics, and modulus of elasticity.

All equipments supports, restraints, and attachments which are not part of the equipment to be analysed, shall be assumed to be rigid. In some cases, the equipment may be mounted in locations for which there are no response spectra available, or which cannot be adequately described by the Floor Response Spectra, such as valves or filters mounted in piping systems or components mounted on other equipment. In these instances, peak accelaration values shall be specified separately for the horizontal and vertical directions, with a statement that the values shown are for rigid equipment or for equipment which is not rigid.

The equipment damping shall be one percent of critical damping for use in seismic analysis.

The earthquake duration shall be considered to be 30 seconds for the Design Basis Earthquake level, and 10 seconds for the Site Design Earthquake level.

For equipment requiring fatigue analysis to comply with an applicable code, the stress at the frequency being analysed shall be considered to act throughout the diration of the earthquake. If the natural frequency is unknown, the stress shall be assumed to act at a frequency of 33 Hz.

The ability of the equipment to satisfy seismic requirement shall be demonstrated by a seismic analysis method which can evaluate the seismic forces' effects.

3.2.2 Methods of seismic analysis

The seismic analysis of the equipment shall evaluate the effects of the seismic forces acting simultaneously with all non-seismic loads to which the equipment will be subjected during normal operation such as gravity, internal pressure and fluid contained in the equipment.

3.2.2.1 Static analysis method

Static analysis is the evaluation of the effect of applying a static load to a specified point on the equipment.

The static load shall be equivalent to the inertia force acting a the centre of gravity of the equipment during the seismic event. The acceleration to be used in the calculation of the static load has to be specified or may be taken as the acceleration shown for the frequency of 33 Hz on the floor response spectra.

Static analysis shall not be used for other than rigid equipment, since amplification occuring in the equipment is not taken into account by this method.

3.2.2.2 Static coefficient method

In the static coefficient method of analysis, the equipment is subjected to a load equal to or greater than that which would exist if the frequency subject to the maximum acceleration, as shown on the floor response spectra, coincides with the dominant natural frequency of the equipment. The actual natural frequencies of the equipment are not determined, but the maximum acceleration shown on the specified floor response spectra is multiplied by an arbitrary factor known as the static coefficient. The static coefficient takes into account multifrequency excitation and multimode response. The static coefficient is normally taken as 1.5.

The seismic forces on each component of the equipment are obtained by multiplying the mass and the maximum acceleration shown on the floor response spectra adjusted by the static coefficient. The seismic force shall be distributed over the equipment in a manner proportional to the mass distribution of the equipment. Equipment which can be represented as a single degree of freedom system may use only the peak value of acceleration shown on the floor response spectra without application of the static coefficient.

3.3 Seismic analysis, applications

3.3.1 Static analysis

According to the specification paragraphs (3.2) and (3.2.2.1), this method can only be used when the vessel or the equipment is considered "rigid". In order to find out if the vessel or the equipment is rigid, the natural frequencies or each main part of the vessel or equipment have to be computed for specified horizontal and vertical directions, and these natural frequencies must be equal to 33 Hz or greater.* If the accelaration to be used with rigid equipment is not specified on the date sheets, the acceleration corresponding to a frequency of 33 Hz on the floor response spectra may be taken.

This equipment damping shall be 1% of critical damping. After having verified that a vessel or piece of equipment is acutally "rigid", all pressure parts, supports, etc. must be subjected to additional loads, lateral and vertical, corresponding to the multiplier specified or to the accelerations at 33 Hz. Stresses computed as a result of such seismic loads must be combined with stresses due to sustained loads such as pressure, weight, etc. Seismic loads resulting from the simultaneous application of several seismic directions need only be combined by taking the square root of the sum of the squares of the individual seismic stress components.

3.3.2 Static analysis, numerical example

An horizontal AIR RECEIVER as shown below, supported on two saddles is required for a nuclear power plant. The horizontal and vertical floor responses spectra for the location are shown on figures (3.3.2), (3.3.3) and (3.3.4). The reference elevation is 71'-0". A damping coefficient

β_e = 1.0% should be used. The shell material is carbon steel SA-516 GR.70 for the vessel and SA-36 for the two saddles. The total operating weight of the vessel is 4200 lb.

Compute a seismic analysis of the air receiver, if the vessel is designed for a category "A" and the operating temperature is 212^{O}F.

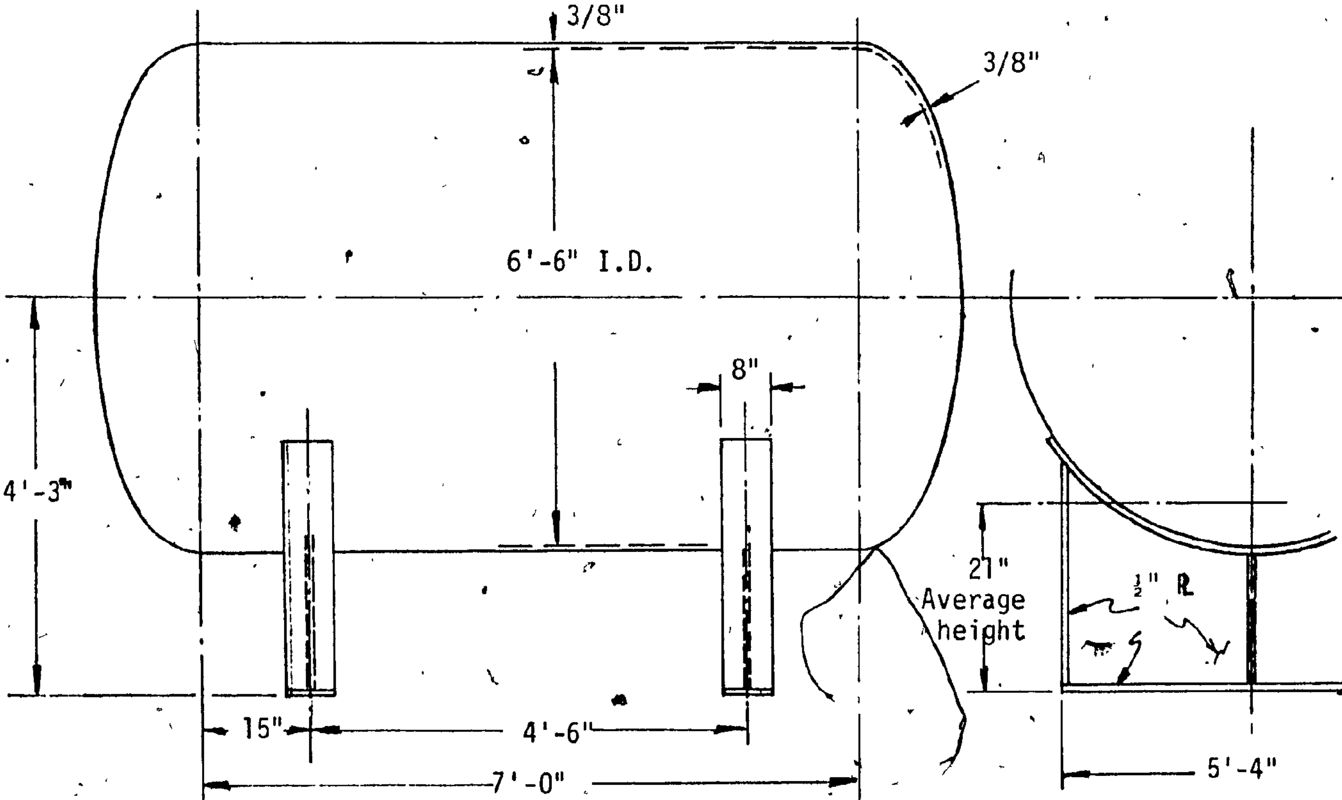


Figure 3.3.1

ELEVATION 71'-0"

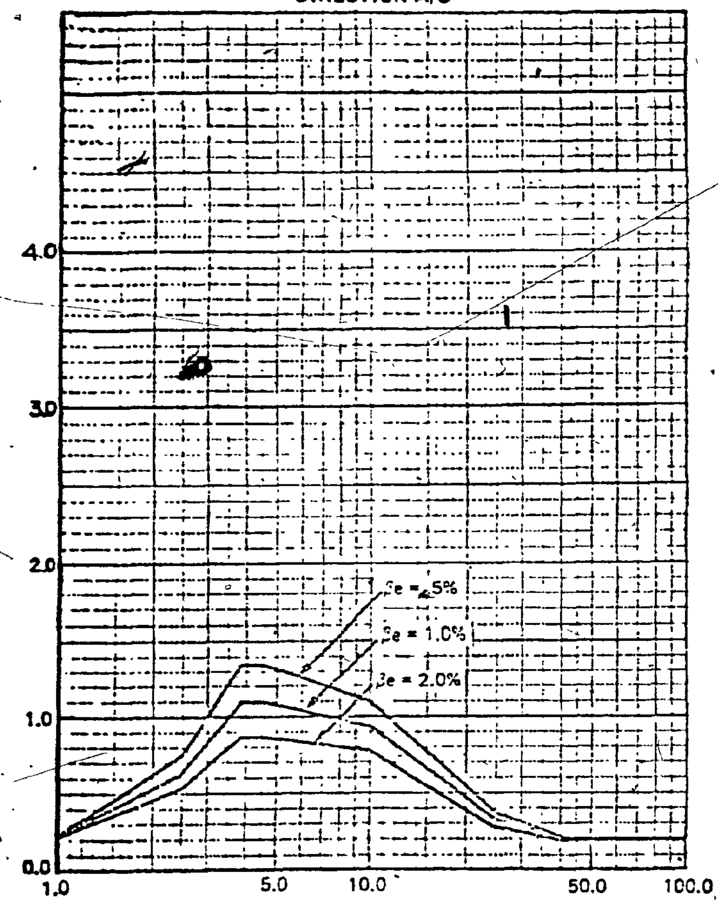


Fig. 3.3.2

NATURAL FREQUENCY OF EQUIPMENT (CPS)

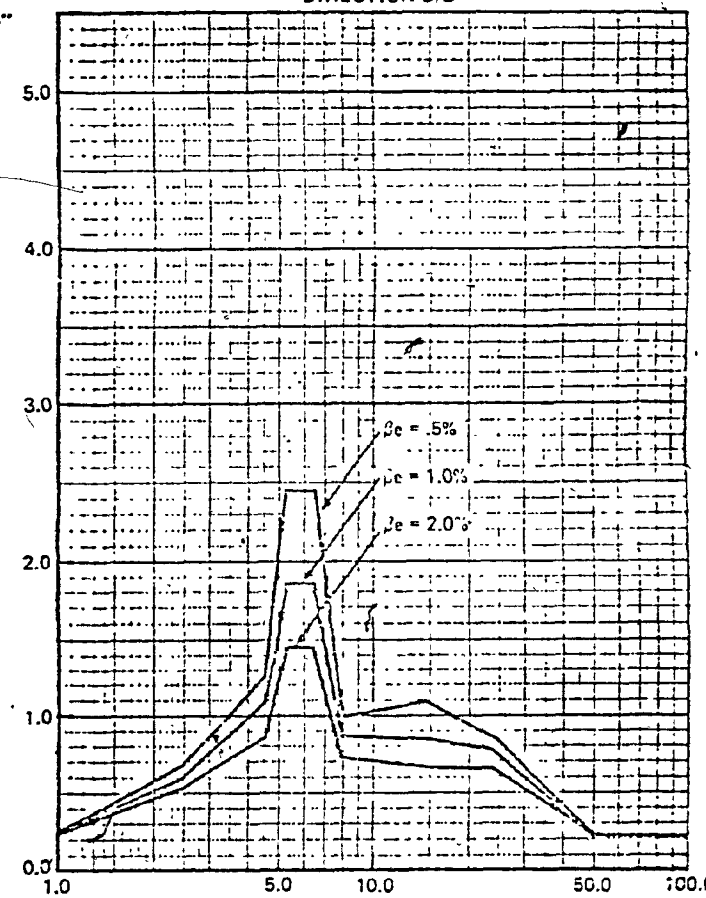


Fig 3.3.3

NATURAL FREQUENCY OF EQUIPMENT (CPS)

~~POINT LEPREAU~~ G.S. - REACTOR BUILDING
HORIZONTAL FLOOR RESPONSE SPECTRA

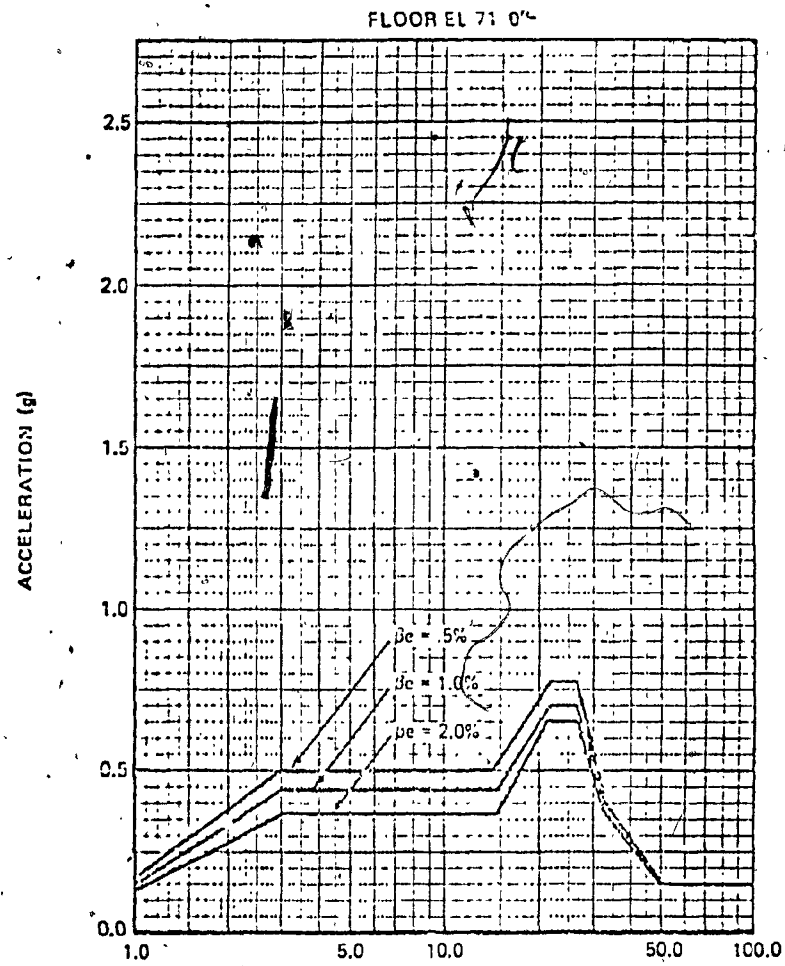


Fig. 3.3.4

NATURAL FREQUENCY OF EQUIPMENT (CPS)

~~POINT-LEPREAU G.S.~~ REACTOR BUILDING
VERTICAL FLOOR RESPONSE SPECTRA

SOLUTION

Moment of inertia of supports

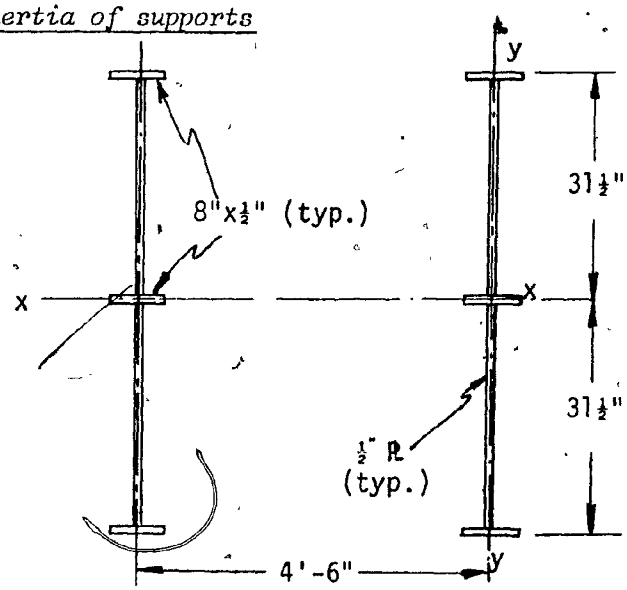


Figure 3.3.5

$$I_x = \frac{(.5)(62^3)}{12} + (2)(4)(31.5^2) = 17870 \text{ in}^4 \text{ (each saddle)}$$

$$I_y = \frac{(3)(.5)(8^3)}{12} = 64 \text{ in}^4 \text{ (each saddle)}$$

$$A = (3)(4) + (2)(15) = 42 \text{ in}^2 \text{ (each saddle)}$$

Natural frequencies

A/C axis, saddles

Assume ½ of total weight applied horizontally to end of cantilever

$$\therefore\ P = \frac{w}{2}$$

$$P = (\tfrac{1}{2})(4200) = 2100 \text{ lb}$$

$$I_x = 17870 \text{ in}^4$$

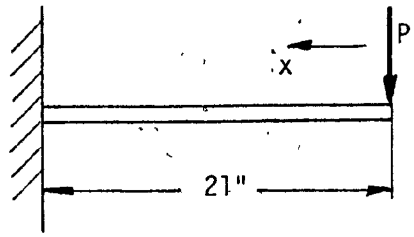


Figure 3.3.6

For a cantilever beam loaded by a force P as shown at one end, and fixed at the other end, the static deflection y_o *is:*

$$y_o = \frac{P\ell^3}{3EI}$$

Natural frequency is

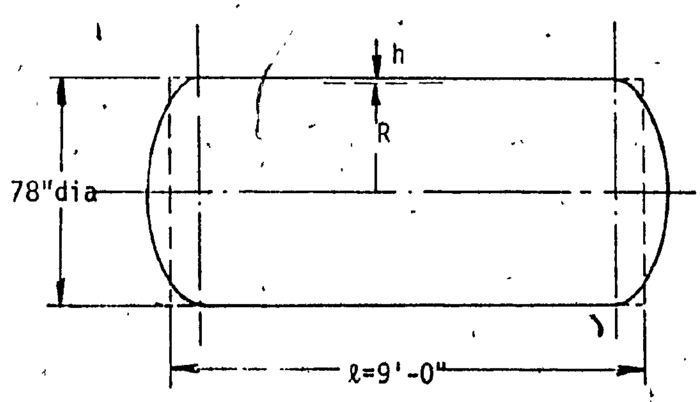
$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{y_o}} = \frac{1}{2\pi}\sqrt{\frac{3gEI}{P\ell^3}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{(3)(386)(29)(10^6)(17870)}{(2100)(21^3)}}$$

$$f_n = 884\ Hz > 33\ Hz$$

A/C axis, cylinder vibrations

Leissa's method (article 2.6) is used. The actual shape of vessel can be approximated by a cylinder with flat ends.



$$\frac{R}{h} = \frac{39}{.375} = 104$$

$$\frac{\ell}{mR} = \frac{108.}{(1)(39)} = 2.77$$

Figure 3.3.7

From fig. 2.6.12, it can be seen that the fundamental frequency occurs when n = 4 (4 radial nodes)

$$\therefore \lambda = \frac{m\pi R}{\ell} = \frac{(1)\pi(39)}{108} = 1.13 \;,\; \frac{h}{R} = 0.010$$

From fig. 2.6.16 (n = 4) for $\lambda = 1.13$ *and* $\frac{h}{R} = 0.010$

$$\Omega = \omega R \sqrt{\rho(1 - \nu^2)/E} = 0.08$$

$$\omega = \sqrt{\frac{\Omega^2 E}{\rho(1 - \nu^2)R^2}}$$

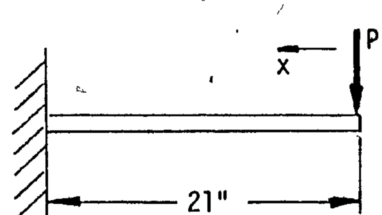
$$\omega = \sqrt{\frac{(386)(0.08^2)(29)(10^6)}{(.283)(1-.3^2)(39^2)}} = 428 \text{ rad/sec}$$

Natural frequency of the cylinder

$$f_n = \frac{\omega}{2\pi} = \frac{428}{2\pi} = 68 \text{ Hz} > 33 \text{ Hz}$$

B/D axis, saddles

Assume the full weight applied horizontally to end of cantilever (fixed saddle)



$P = W$

$P = 42000$ lb

$I_y = 64$ in³

Figure 3.3.8

For a cantilever beam loaded by a force P at one end, and fixed at the other end, the static deflection y_o *is:*

$$y_o = \frac{P\ell^3}{3EI_y}$$

Natural frequency is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{g}{y_o}} = \frac{1}{2\pi}\sqrt{\frac{3gEI_y}{P\ell^3}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{(3)(386)(29)(10^6)(64)}{(4200)(21^3)}}$$

$$f_n = 37\ Hz > 33\ Hz$$

B/D axis, cylinder vibrations

Shell vibrations produce frequencies much higher than saddle vibrations.

Vertical axis, cylinder vibrations

The natural frequency for this axis is the same as in case of A/C axis cylinder.

Vertical axis, saddles

For a bar of uniform cross section loaded by a force "P" at one end, the other being fixed, the natural frequency is given by

$$f_n = \frac{1}{2\pi}\sqrt{\frac{kg}{P}}$$

where spring stiffness $k' = \frac{EA}{\ell}$

$$A = (3)(4) + (2)(15.5) = 43\ in^2$$

$$\therefore f_n = \frac{1}{2\pi}\sqrt{\frac{gEA}{P\ell}}$$

$$f_n = \frac{1}{2\pi}\sqrt{\frac{(386)(29)(10^6)(43)}{(2100)(21)}}$$

$$f_n = 526\ Hz > 33\ Hz$$

Seismic analysis (Static analysis per 3.2.2.1)

All natural frequencies are found greater than 33 Hz, therefore the vessel can be considered rigid and the static analysis method may be used.

All acceleration will be taken at 33 Hz for a more conservative approach.

A) *Horizontal axis*

1) *A/C axis, saddles*

From fig. (3.3.2) the maximum lateral acceleration at 33 Hz, is .26 g.

$$F = (.26)(W) = (.26)(4200) = 1092 \text{ lb}$$

$$V_b = \tfrac{1}{2}F = \frac{1092}{2} = 546 \text{ lb}$$

$$M = V_b s = (546)(51)$$

$$M = 27850 \text{ in-lb}$$

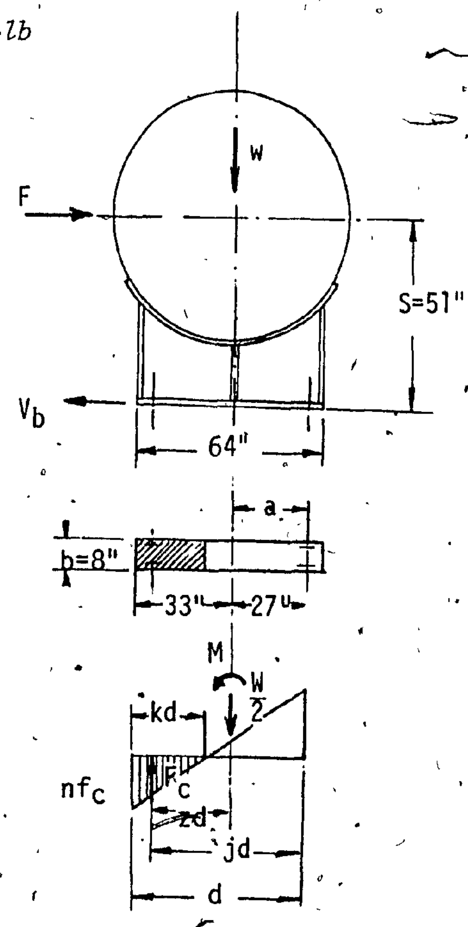


Figure 3.3.9

There are 4-1" Bolts per saddles, we assume 2 Bolts effective

$$A_s = (2)(.785) = 1.570 \text{ in}^2$$

Using ref. [16]

$$p_n = \frac{nA_s}{bd} = \frac{(10)(1.570)}{(8)(60)} = .0327$$

$$\frac{e}{d} = \frac{M}{W/2d} + \frac{a}{d} = \frac{27850}{(2100)(60)} + \frac{27}{60} = .671$$

$$k^3 + 3\left(\frac{e}{d} - 1\right)k^2 + 6p_n \frac{e}{d} k = 6\, p_n \frac{e}{d}$$

$$k^3 - .987\, k^2 + .1316\, k = .1316$$

$$k = .880\ , \quad j = 1 - \frac{k}{3} = .707$$

$$f_c = \frac{2(W/2)(e/d)}{bdkj} = \frac{(2)(2100)(.671)}{(8)(60)(.880)(.707)} = 9.5 \text{ psi}$$

<u>Base plate stress</u>

$$M = \frac{f_c \ell^2}{2} = \frac{(9.5)(3.75^2)}{2} = 66.8 \text{ in-lb/in}$$

$$S_b = \frac{6M}{t^2} = \frac{(6)(66.8)}{(.50^2)} = 1610 \text{ psi}$$

<u>Saddle bending stress</u>

$$M = \frac{F_s}{2} = (546)(51)$$

$$M = 27850 \text{ in-lb}$$

$$S_b = \frac{W/2}{A} + \frac{My}{I_x}$$

$$S_b = \frac{2100}{42} + \frac{(27850)(32)}{17870}$$

$$S_b = 50 + 50 = 100 \text{ psi}$$

2) *B/D axis, saddles*

From fig. (3.3.3) the maximum lateral acceleration at 33 Hz is .53 g

$$F = (.53)(W) = (.53)(4200) = 2226 \text{ lb}$$

$$V_b = 2226 \text{ lb (one saddle has slotted holes)}$$

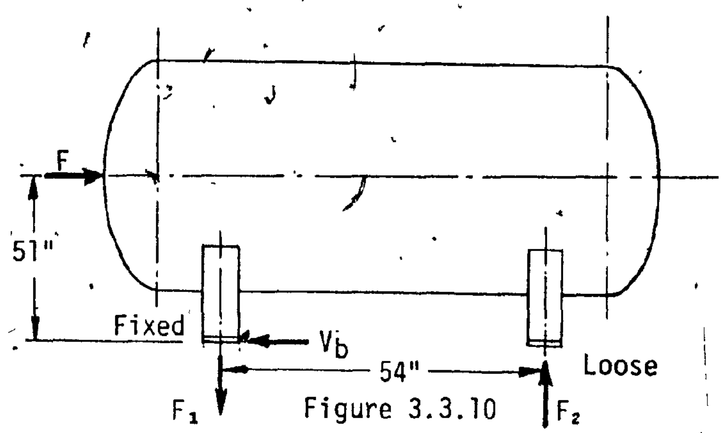


Figure 3.3.10

Saddle force reactions

$$F_1 = F - \tfrac{1}{2}W = 2226 - 2100 = 126 \text{ lb (uplift)}$$

$$F_2 = F + \tfrac{1}{2}W = 2226 + 2100 = 4326 \text{ lb}$$

Base plate stress

$$p = \frac{F_2}{A_1} = \frac{4326}{(64)(8)} = 8.4 \text{ psi}$$

$$M = \frac{p\ell^2}{2} = \frac{(8.4)(3.75^2)}{2} = 59.1 \text{ in-lb/in}$$

$$S_b = \frac{6M}{t^2} = \frac{(6)(59.1)}{(.50^2)} = 1420 \text{ psi}$$

Saddle bending stress

$$M = F\,s = (2226)(51) = 113526 \text{ in-lb}$$

$$S_b = \frac{W/2}{A} + \frac{M_y}{I_y} = \frac{2100}{42} + \frac{(113526)(4)}{64}$$

$$S_b = 50 + 7095 = 7145 \text{ psi}$$

B) <u>Vertical axis</u>

From fig. (3.3.4) the maximum vertical acceleration at 33 Hz is .38 g

$$Q = (.38)(\frac{W}{2}) = (.38)(\frac{4200}{2}) = 800 \text{ lb per each saddle}$$

$$P = \frac{1}{2}W + Q = 2100 \pm 800 = \begin{cases} +2900 \\ +1300 \end{cases} \text{lb}$$

<u>Base plate stress</u>

$$p = \frac{P}{A_1} = \frac{2900}{(64)(8)} = 5.7 \text{ psi}$$

$$M = \frac{p\ell^2}{2} = \frac{(5.7)(3.75^2)}{2}$$

$$M = 40.1 \text{ in-lb/in}$$

$$S_b = \frac{6M}{t^2} = \frac{(6)(40.1)}{(.50)^2} = 960 \text{ psi}$$

<u>Saddle stress</u>

$$S_c = \frac{P}{A} = \frac{2900}{42} = 69 \text{ psi}$$

C) <u>Combined stresses</u> for the 3 accelerations directions

1) <u>Base plate</u>

$$S_t = \sqrt{1610^2 + 1420^2 + 960^2}$$

$S_t = 2350$ psi < 14500 pis (A-36 allowable stress)

2) <u>Saddles</u>

$$S_t = \sqrt{100^2 + 7145^2 + 69^2}$$

$S_t = 7146$ psi < 14500 psi (SA-36)

Note: Attachment stresses due to horizontal and vertical accelerations were also computed, but are not shown in this report. These computations are normally made in accordance with ref. [11].

3.3.3 Static coefficient method

According to the specification paragraphs (3.2) and (3.2.2.2), this method must be used when the actual natural frequencies of the vessel or the equipment cannot be determined. Alternately, for light, strong vessels or equipment, it may be advantageous to forego lengthy frequency calculations in favor of higher seismic forces.

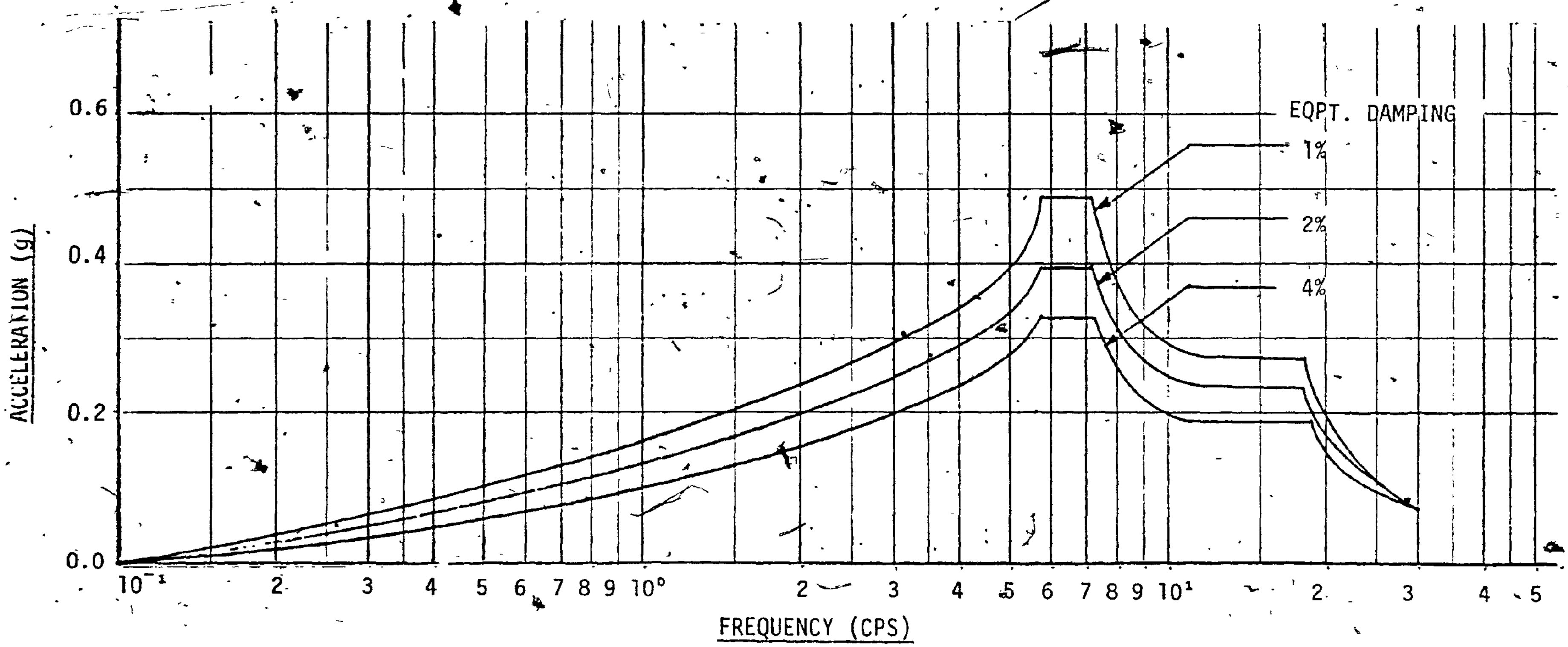
- The equipment damping shall be 1% of critical damping.

From given floor response spectra, the maximum acceleration with 1% critical damping is multiplied by a static coefficient of 1.5. The seismic forces on each component of the vessel or equipment are obtained in horizontal and vertical directions by multiplying the mass by the maximum acceleration shown on the floor response spectra, multiplied by the static coefficient.

Horizontal and vertical stress components are then computed, and finally, combined stresses are obtained, by taking the square root sum of squares of the stresses for each component analyzed.

3.3.4 Static coefficient method, numerical example

A vertical condensate tank for a nuclear power station as shown below is supported on four legs (6"-SCH. 80 pipe). The horizontal and vertical floor responses spectra are shown in figures (3.3.12) to (3.3.17). The reference elevation is 615'-0". A damping coefficient β_e = 1.0% should be used. The tank material is stainless steel SA-240 T-304 L, and legs are SA-53, GR.B pipes with carbon steel base plates. The total operating weight of the vessel is 6000 lb. Compute a seismic analysis for the vertical con-

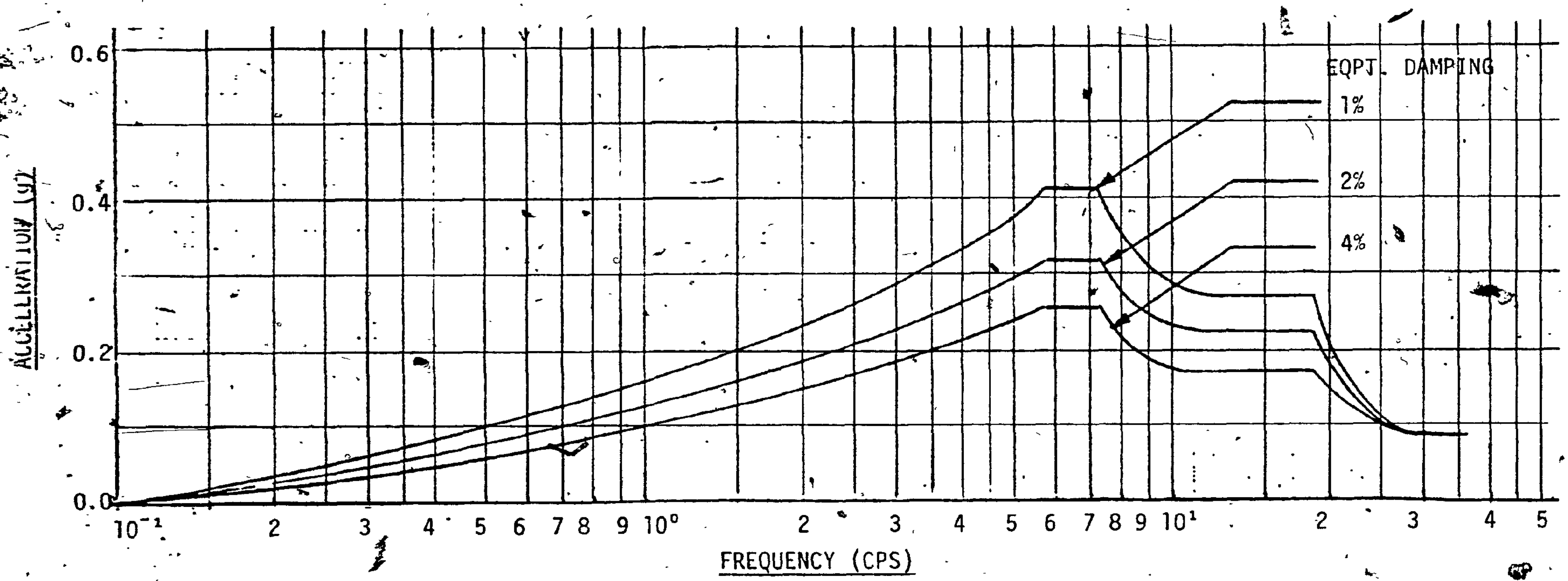


Equipment damping = 1%

Interpolate corresponding "g" value for el. 615'-0"

Condensate tank
Bruce G.S.B.
Reactor building
N.S. horizontal floor response spectra
Floor elev. 612'-0" (node 5 of model)

FIGURE 3.3.12

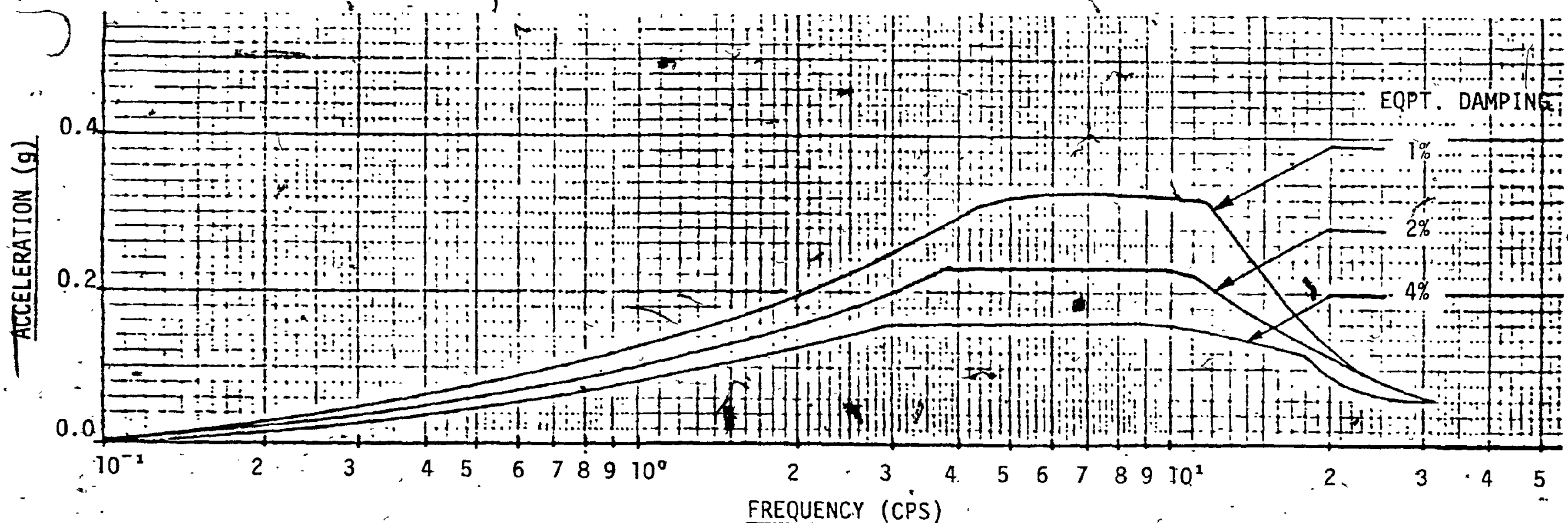


Equipment damping = 1%

Interpolate corresponding "g" value for el. 615'-0"

Condensate tank
Bruce G.S.B.
Reactor building
E.W. horizontal floor response spectra
Floor elev. 612'-0" (node 5 of model)

FIGURE 3.3.13

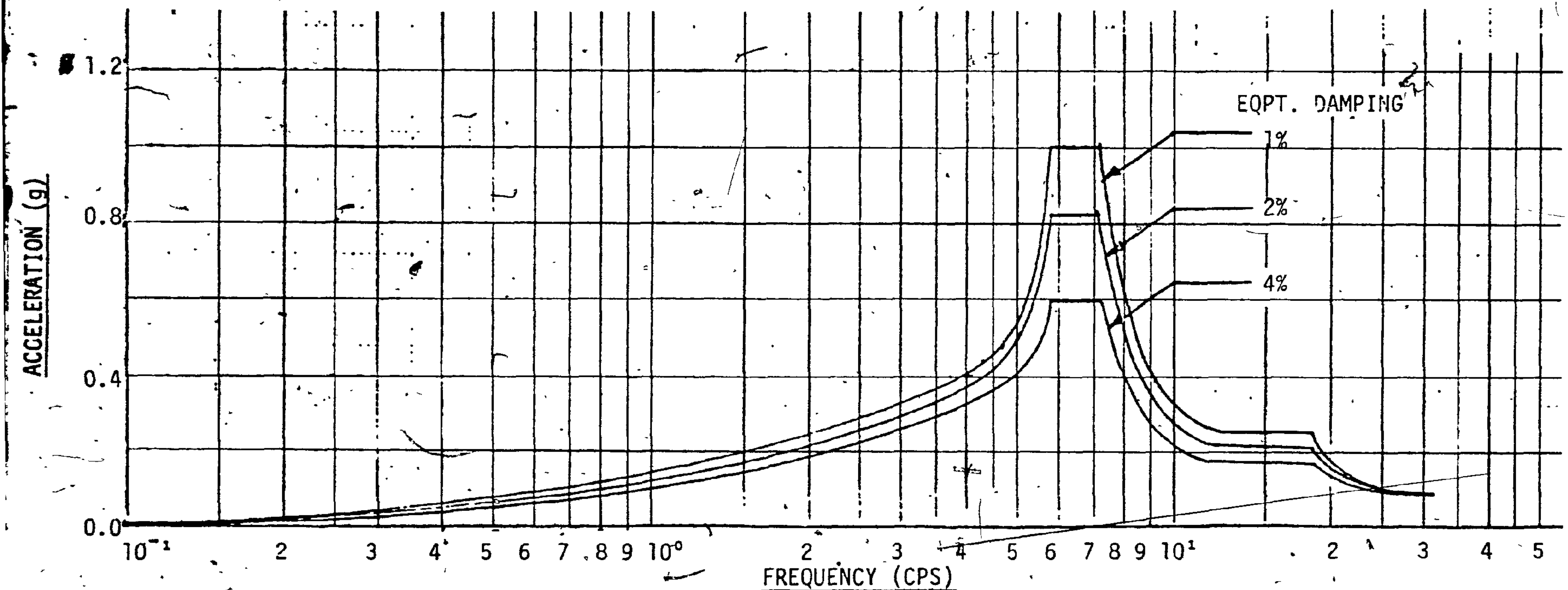


Equipment damping = 1%

Interpolate corresponding "g" value for el. 615'-0"

Condensate tank
Bruce G.S.B.
Reactor building
Vertical floor response spectra
Floor elev. 612'-0" (node 5 of model)

FIGURE 3.3.14

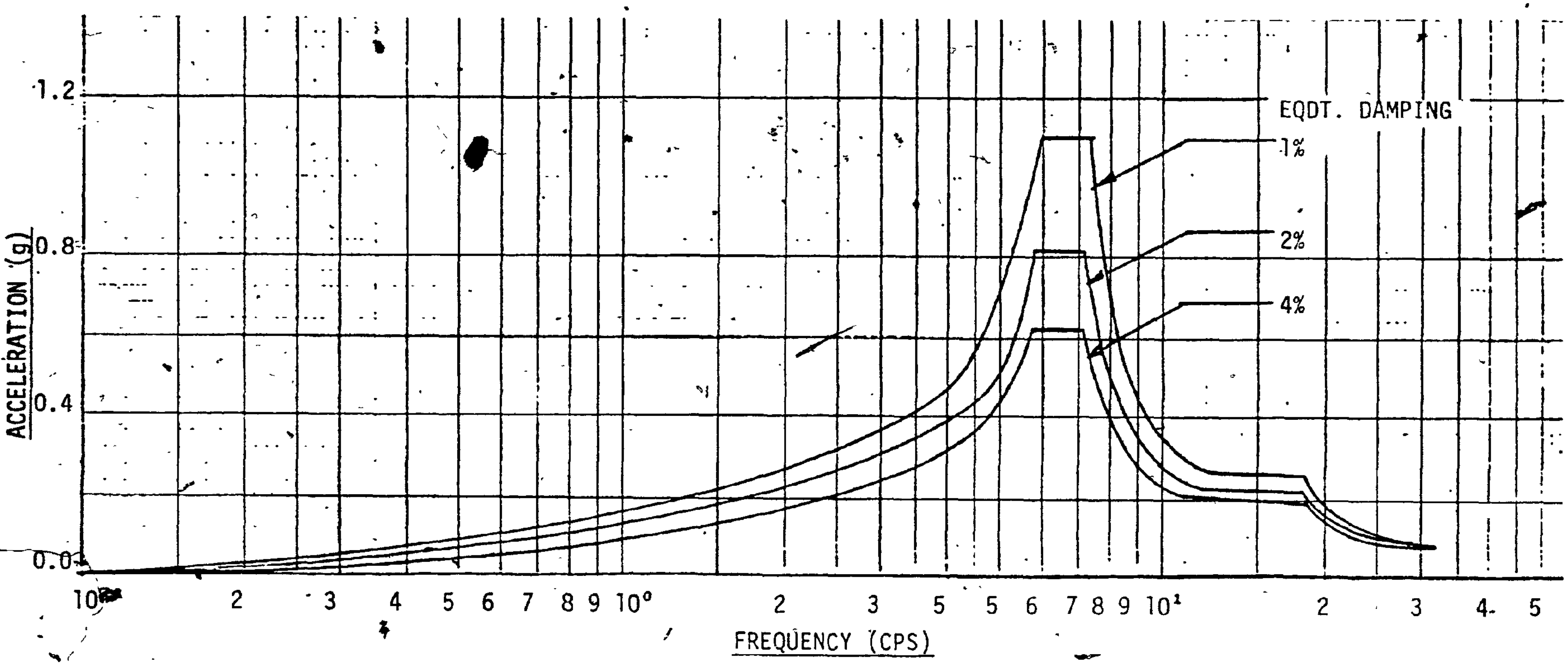


Equipment damping = 1%

Interpolate corresponding "g" value for el. 615'-0"

Condensate tank
Bruce G.S.B.
Reactor building
E.W. horizontal floor response spectra
Floor elev. 639'-0" (node 6 of model)

FIGURE 3.3.15

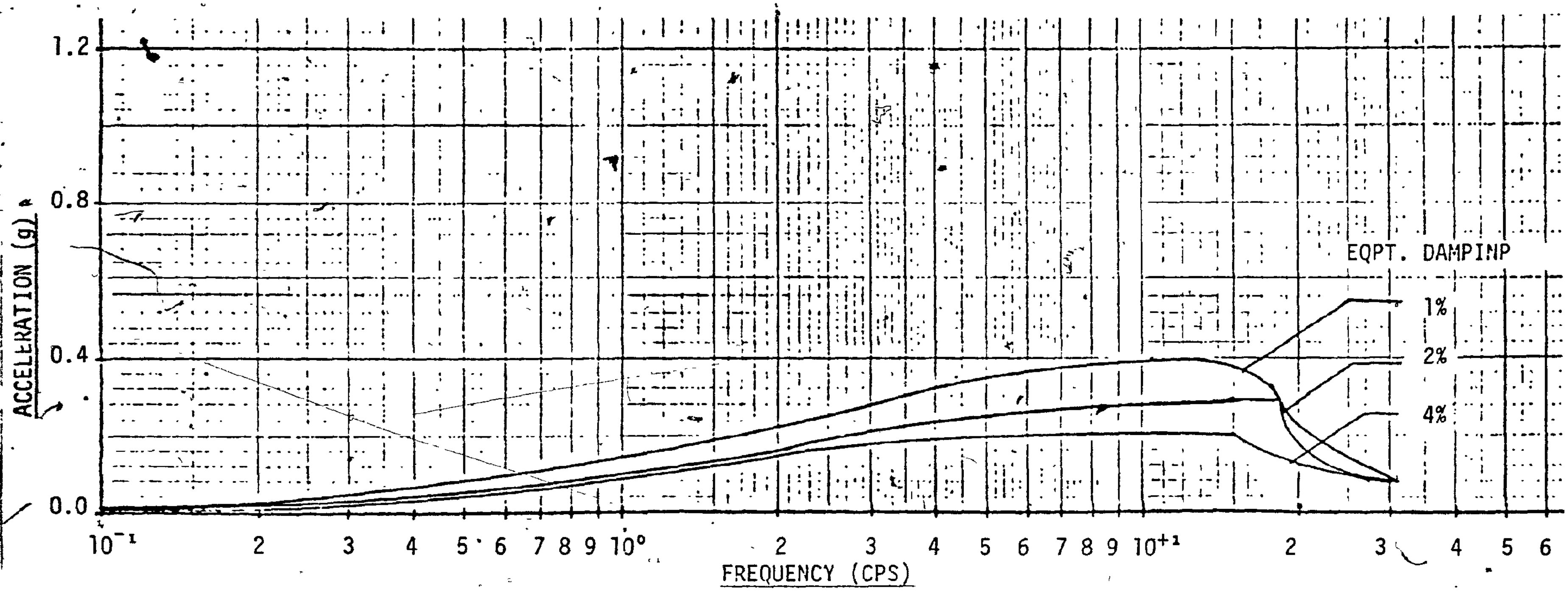


Equipment damping = 1%

Interpolate corresponding "g" value for elev. 615'-0"

Condensate tank
Bruce G.S.B.
Reactor building
N.S. horizontal floor response spectra
Floor elev. 639'-0" (node 6 of model)

FIGURE 3.3.16



Equipment damping = 1%

Interpolate corresponding "g" value for el. 615'-0"

Condensate tank
Bruce G.S.B.
Reactor building
Vertical floor response spectra
Floor elev. 639'-0" (Node 6 of model)

FIGURE 3.3.17

densate tank, if the vessel is designed for a category "A" and the operating temperature is 150^oF.

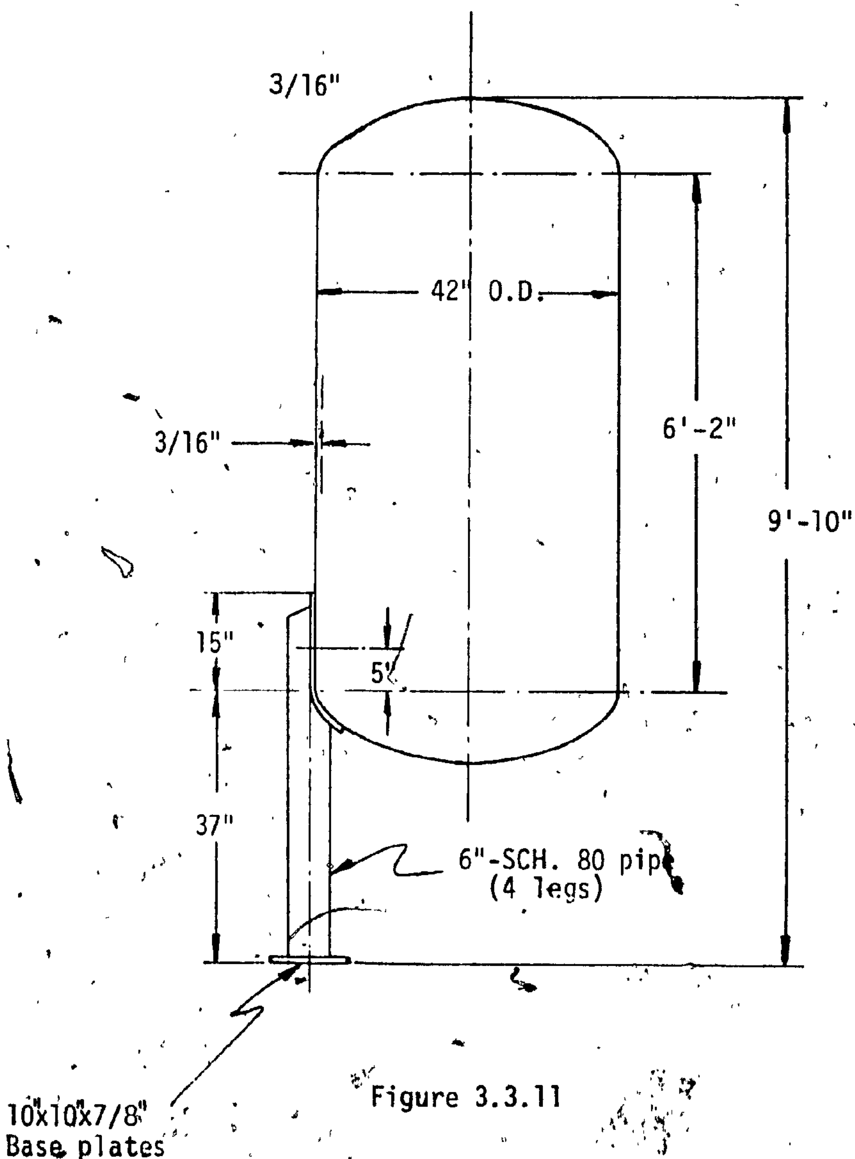


Figure 3.3.11

SOLUTION

The static coefficient method is chosen; computation of natural frequencies of the vessel components is not necessary.

From figures 3.3.12 and 3.3.15 (N.S. horizontal direction) using a 1% damping coefficient and interpolating for required elevation 615'-0", the maximum horizontal N.S. acceleration is found to be .48 g.

From figure 3.3.13 and 3.3.16 (E.W. horizontal direction) using a 1% damping coefficient and interpolating elevation 615'-0", the maximum horizontal E.W. acceleration is found to be .64 g.

From figure 3.3.14 and 3.3.17 (vertical direction) using a 1% damping coefficient and interpolating for elevation 615'-0", the maximum vertical acceleration is found to be .31 g.

Using a static coefficient of 1.5 the following maximum seismic forces are obtained:

N.S. horizontal direction

$$F = (.48)(1.5)(6000) = 4320 \text{ lb}$$

E.W. horizontal direction

$$F = (.64)(1.5)(6000) = 5760 \text{ lb}$$

Vertical direction

$$Q = (.31)(1.5)W$$

$$Q = (.31)(1.5)(6000)$$

$$P = W \pm Q$$

$$P = 6000 \pm (.31)(1.5)(6000) = 6000 \pm 2790$$

$$P = \begin{cases} +8790 \\ +3210 \end{cases} \text{lb}$$

A) *Support legs*

1) *N.S. Horizontal load*

$$F = 4320\ lb$$

$$V_b = \frac{4320}{4} = 1080\ lb$$

$$W = 6000\ lb$$

$$F_1 = \frac{(4320)(74) + (6000)(14.85)}{(4)(14.85)} = 6882\ lb$$

$$F_2 = \frac{(4320)(74) - (6000)(14.85)}{(4)(14.85)} = 3882\ lb$$

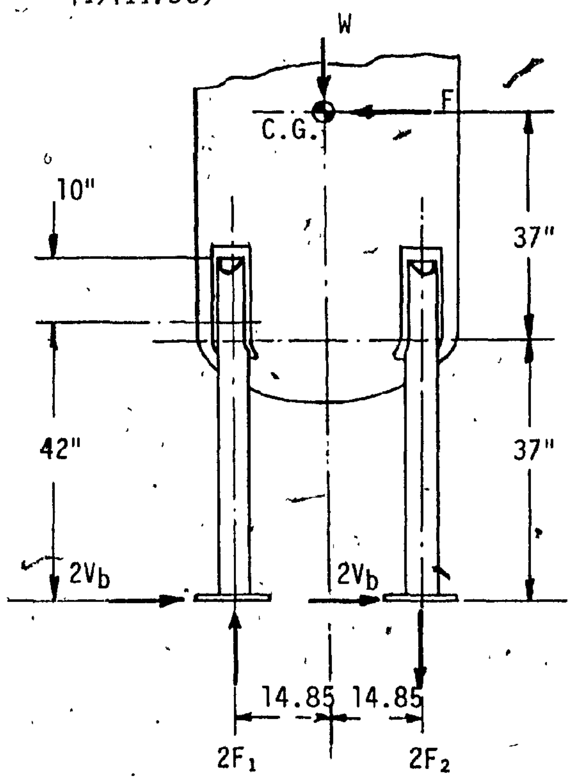


Figure 3.3.18

2) *W.E. horizontal load*

$F = 5760\ lb$

$V_b = \frac{5760}{4} = 1440\ lb$

$W = 6000\ lb$

$F_2 = (6000)(\frac{1}{4}) = 1500\ lb$

$F_1 = \frac{(5760)(74) + (3000)(21)}{(2)(21)}$

$F_1 = 11648\ lb$

$F_3 = \frac{(5760)(74) - (3000)(21)}{(2)(21)}$

$F_3 = 8648\ lb$

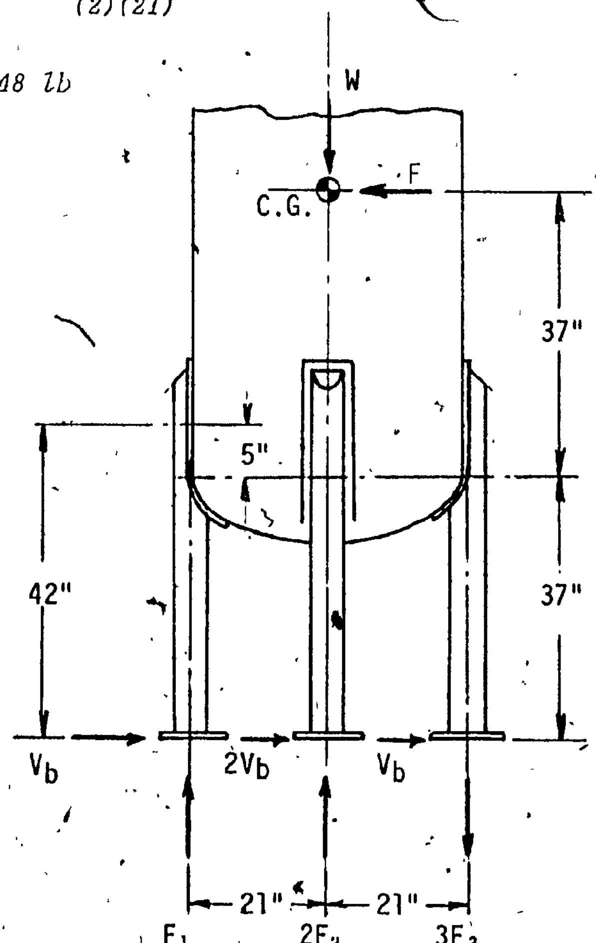


Figure 3.3.19

4.1.5 Foundation factor, F_b

F_b = 1.0 for rock, dense and very dense coarse-grained soils. Vessels are usually sitting on a concrete base.

F_b = 1.5 for loose and loarse grained soils.

4.1.6 Weight of vessel, W

The weight, W, of the vessel shall be the operating weight.

4.2 Distribution of lateral seismic force

The total lateral seismic force, V_b, shall be distributed as follow:

$$V_b = F_t + F_e \tag{4.5}$$

a) A portion F_t shall be assumed to be concentrated at the top of the vessel.

$$F_t = 0.15\ V_b \tag{4.6}$$

b) The remainder,

$$F_e = V_b - F_t \tag{4.7}$$

shall be distributed along the height of the vessel.

c) Distributed load w

$$w = \frac{F_e}{L} \tag{4.8}$$

4.3 Overturning moment

The overturning moment at any level, x, M_x, is the product of the lateral force applied at top, F_t, times the height at level x, plus the moment created by the distributed load w, (see fig. 4.2).

$$M_x = F_t x + \frac{wx^2}{2} \tag{4.9}$$

The overturning moment, M, at the base of a pressure vessel shall be multiplied by a reduction coefficient, J, where:

a) J = 1 where T is less than 0.5

b) J = (1.1 - 0.2 T) where T is at least 0.5, but not more than 1.5

c) J = 0.8 where T is greater than 1.5

$$\therefore \quad M = [F_t L + \frac{wL^2}{2}]\, J \tag{4.10}$$

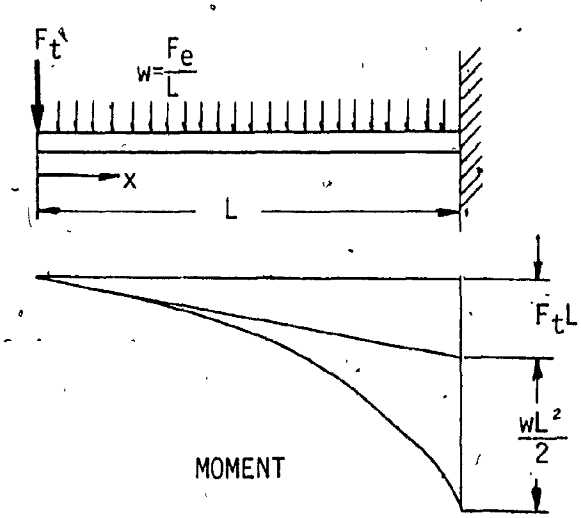


Figure 4.2

Example 4.1 A fractionator tower is 36" diameter and 60 feet between tangent lines. It is supported on a 12 feet high skirt. The design conditions are 60 psig and 350°F. The tower is erected in a earthquake zone 2. The fractionator weighs 40,000 lbs empty and 64,000 lbs when operating. Find the overturning moment at base of the tower.

SOLUTION

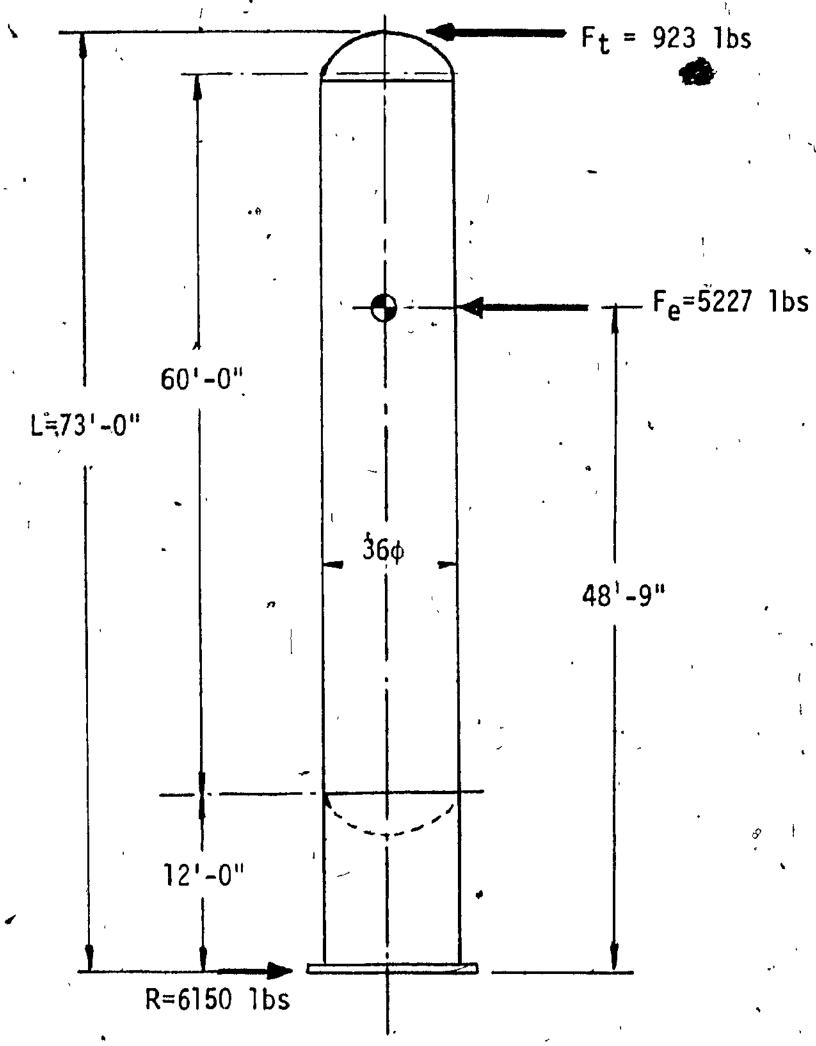


Figure 4.1.1

Effect of earthquake

NBC of Canada 1977, commentary J on part 4

Lateral seismic force, V_b

$$V_b = A_r S K I_b F_b W$$

Acceleration ratio, A_r

$A_r = 0.04$ (Table 4.1, seismic zone 2)

Seismic coefficient, S

$$S = \frac{0.5}{\sqrt[3]{T}}$$

$$T = \frac{0.05L}{\sqrt{D}} = \frac{0.05\ (73)}{\sqrt{3}} = 2.107$$

$$\therefore\ S = \frac{0.5}{\sqrt[3]{2.107}} = \frac{0.5}{1.282} = .390$$

Numerical coefficient, K

$K \doteq 2.0$

Importance factor, I_b

$I_b = 1.0$

For pressure vessels

$SKI_b = 2.5$ max and

$= 1.2$ min

$SKI_b = (.390)(2.0)(1.0) = .78$

$\therefore$ take $SKI_b = 1.2$

Fondation factor, F_b

$F_b = 1.0$

Operating weight, W

$W = 64{,}000$ lbs

$\therefore\ V_b = (0.04)(1.2)(2.0)(1.0)(1.0)(64000)$

V_b = 6144 lbs, say 6150 lbs

Lateral forces distribution

$V_b = F_t + F_e$

$F_t = 0.15\ V_b = 0.15\ (6150) = 923$ lbs at top

$F_e = 0.85\ (6150) = 5227$ lbs

Overturning moment, M at the base of the tower

$$w = \frac{5227}{73} = 71.6\ \frac{lb}{ft}$$

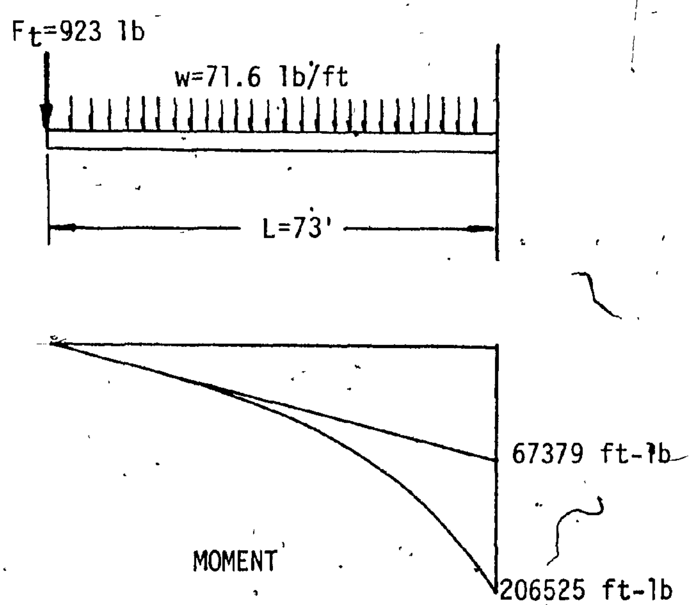


Figure 4.1.2

$$M = [F_t L + \frac{wL^2}{2}]\ J$$

where $J = 0.8$ *since* T *is greater than 1.5*

$$M = [923(73) + \frac{71.6(73^2)}{2}]\ (0.8)$$

$$M = [67379 + 190778]\ (0.8) = \underline{\underline{206525}}\ \text{ft-lb}$$

5.0 CONCLUSION

The seismic analysis methods presented in this report were mainly selected due to their simple applications, and could be very well used by the designer of nuclear power plant equipments and the designer of special structures, found in the petroleum industry such as, towers, pressure vessels and chimneys.

Part 2.0 of this report describes various methods for the computation of the natural frequencies of pressure vessels for several end support conditions. In general, the computation of natural frequencies and mode shapes of a pressure vessel is a very lengthy operation, and unless a very precise solution is required, the data presented by Leissa's work is in a format which can be used directly by a designer. The advantages of this method is that a pressure vessel can be approximated by a cylinder with flat ends, and by using the graphs presented for the proper boundary conditions, the frequency parameter Ω can be found and the fundamental frequency of the cylinder computed very rapidly.

Several examples are covered in this report in using this method, and if we compare example 2.1, which illustrates the lateral vibrations of a cantilever beam, with example 2.7 which uses Leissa's work, we can see that the natural frequencies are not the same, but are very close from each other. This indicates that Leissa's work is an acceptable and conservative method to find the lowest natural frequency of a pressure vessel.

The example 2.2 using the differential equation method for lateral vibration of beam can be compared to example 2.4, which illustrated Rayleigh's method for the computation of the natural frequencies. The results show that the two answers are almost identical, which indicates a good accuracy of the two methods.

Part 3.0 of this report describes the seismic analysis of nuclear power plant equipment, using two simplified methods:

- static analysis
- static coefficient analysis

The above two methods are acceptable by AECL and have a potential application to the design of pressure vessels of categories "A" and "B" found in the industry.

The static analysis is lengthy to execute than the static coefficient analysis, since the computation of all natural frequencies of the equipment is required. This is to ensure that the lowest frequency is found greater or equal to 33 Hz, which indicates that the equipment can be considered rigid.

The static coefficient analysis is used whenever the natural frequencies of the equipment cannot be determined. However, for light, strong vessels or equipment, it may be advantageous to compute lengthy frequency calculations in favor of higher seismic forces, since this induces an over designed equipment, which represents several additional costs.

Part 4.0 of this report describes the seismic analysis of buildings and industrial installations, using a static load analysis. The National Building Code of Canada has first introduced this method, which the primary interest of such codes, is the design and construction of buildings. However, the NBC of Canada is of a broader scope and has criteria included for structures other than buildings such as towers and chimneys.

The advantages of this method are the following;

- it is simple to follow and to compute
- it can be executed rapidly
- it gives to the designer a preliminary estimate for the evaluation of the sizes of the structure.

However, it is beyond the scope of the NBC to cover the entire range of problems involved in the seismic design of structures such as nuclear reactors. These structures should be treated as special problems with special design criteria in each instance, including possibly a computer dynamic analysis.

We can arrive to the final conclusion that the above three mentioned methods have some advantages and some inconvenients, and they have to be used with good judgement and precaution, depending on the importance and the safety of the equipment to be utilized.

REFERENCES

[1] Blume, Newmark and Corning - Design of Multistory Building for Earthquake motions, Portland Cement Association, 1961

[2] Proceedings - Second Canadian Conference on Earthquake Engineering. 1975

[3] Weigel L. Robert, Earthquake Engineering, Englewood Clifes N.H., Prentice-Hall Inc., 1970

[4] William T. Thomson, Theory of Vibration with Applications, Prentice-Hall Inc., 1972

[5] Shigley E. Joseph, Dynamic Analysis of Machines, Toronto, McGraw-Hill, 1961

[6] A.F. Black, Process Equipment Design Course, unpublished Lecture Notes, 1976

[7] Fertis D.G. and Zobel E.C., Transverse Vibration Theory, The Ronald Press Company, New-York, 1961

[8] A.E.C.L., Technical Specification TS-XX-30000-5 "Seismic Qualification of equipment", Nov. 1975 rev. 1

[9] NRC of Canada, National Building Code of Canada, 1977

[10] NRC of Canada, Commentaries on part 4 of the National Building Code of Canada, 1977